1973-09

Prediction of airplane steady spin conditions by a parameter optimization scheme

Keith, Stephen Thomas

Monterey, California: Naval Postgraduate School

http://hdl.handle.net/10945/16749

PREDICTION OF AIRPLANE STEADY SPIN CONDITIONS BY A PARAMETER OPTIMIZATION SCHEME

Stephen Thomas Keith

NAVAL POSTGRADUATE SCHOOL
Monterey, California

THESIS

PREDICTION OF AIRPLANE STEADY SPIN
CONDITIONS BY A
PARAMETER OPTIMIZATION SCHEME

by

Stephen Thomas Keith

Thesis Advisor: M. H. Redlin

September 1973

Prediction of Airplane Steady Spin
Conditions by a
Parameter Optimization Scheme

by

Stephen Thomas Keith
Lieutenant, United States Naval Reserve
B.S., Boston University, 1966

Submitted in partial fulfillment of the
requirements for the degree of

MASTER OF SCIENCE IN AERONAUTICAL ENGINEERING

from the

NAVAL POSTGRADUATE SCHOOL
September 1973

ABSTRACT

To aid in the modeling of a steady state spin, the equations of motion of an airplane are formulated in a cylindrical coordinate reference frame. The derivation of the equations is presented and the resulting equations are simplified for the equilibrium spin condition. These simplified equations are used in an unconstrained computer parameter optimization technique that algebraically solves the differential equations for the equilibrium state. The results of the computer work are presented and compared with previous prediction schemes. The potential of the method is demonstrated by application to a study of the effects of density variation.

TABLE OF CONTENTS

LIST OF TABLES

LIST OF FIGURES

LIST OF SYMBOLS

The definitions of the symbols used in this paper are as follows:

b	Wing span, ft
$\bar{c}$	Mean aerodynamic chord, ft
F_{q_r}	Lagrange generalized force, lbs or ft-lbs
F_X, F_Y, F_Z	Aerodynamic forces along the body axes, lbs
g	Acceleration of gravity, ft/sec^2
I_X, I_Y, I_Z, I_{XZ}	Body axes moments and products of inertia, slugs-ft^2
M_X	Rolling moment acting about the X body axis, ft-lbs
M_Y	Pitching moment acting about the Y body axis, ft-lbs
M_Z	Yawing moment acting about the Z body axis, ft-lbs
m	Airplane mass, slugs
p, q, r	Angular rates of rotation about the X, Y and Z body axes, rad/sec
q_∞	Dynamic pressure, $1/2\ \rho V^2$
q_r	Lagrange generalized coordinate
R	Radius of helical path of aircraft, ft
S	Wing area, ft^2
T	Lagrange kinetic energy, ft-lbs
V	Inertial velocity of the C.G., ft/sec
v	Lagrange potential energy, ft-lbs
X, Y, Z	Aircraft body axes

X_1, Y_1, Z_1 — Cartesian coordinate reference axes that are attached to the cylindrical radius vector.

Z_o — Aircraft altitude, ft

α — Angle of attack, degrees

α_{ij} — Direction cosine, dimensionless

β — Angle of sideslip, degrees

γ — Cylindrical orientation angle, radians

δ_a — Aileron deflection (positive when right aileron trailing edge is down), degrees

δ_e — Elevator deflection (positive when elevator trailing edge is down), degrees

δ_r — Rudder deflection (positive when rudder trailing edge is left when viewed from above), degrees

ρ — Atmospheric density, slugs/ft^3

ψ, θ, ϕ — Euler orientation angles

$(\dot{\ })$ — Derivative with respect to time

Coefficients and derivatives:

$$C_\ell = \frac{M_X}{q_\infty S b} \qquad C_m = \frac{M_Y}{q_\infty S b} \qquad C_n = \frac{M_Z}{q_\infty S b}$$

$$C_{\ell_{\delta_a}} = \frac{\partial C_\ell}{\partial \delta_a} \qquad C_{m_{\delta_e}} = \frac{\partial C_m}{\partial \delta_e} \qquad C_{n_{\delta_a}} = \frac{\partial C_n}{\partial \delta_a}$$

$$C_{\ell_{\delta_r}} = \frac{\partial C_\ell}{\partial \delta_r} \qquad C_{m_q} = \frac{\partial C_m}{\partial (\frac{q\bar{c}}{2V})} \qquad C_{n_{\delta_r}} = \frac{\partial C_n}{\partial \delta_r}$$

$$C_{\ell_p} = \frac{\partial C_\ell}{\partial (\frac{pb}{2V})} \qquad C_{n_p} = \frac{\partial C_n}{\partial (\frac{pb}{2V})}$$

$$C_{\ell_r} = \frac{\partial C_\ell}{\partial (\frac{rb}{2V})} \qquad C_{n_r} = \frac{\partial C_n}{\partial (\frac{rb}{2V})}$$

$$C_X = \frac{F_X}{q_\infty S}$$

$$C_{X_{\delta_e}} = \frac{\partial C_X}{\partial \delta_e}$$

$$C_{X_{\dot{q}}} = \frac{\partial C_X}{\partial (\frac{q\bar{c}}{2V})}$$

$$C_Y = \frac{F_Y}{q_\infty S}$$

$$C_{Y_{\delta_a}} = \frac{\partial C_Y}{\partial \delta_a}$$

$$C_{Y_{\delta_r}} = \frac{\partial C_Y}{\partial \delta_r}$$

$$C_{Y_p} = \frac{\partial C_Y}{\partial (\frac{pb}{2V})}$$

$$C_{Y_r} = \frac{\partial C_Y}{\partial (\frac{rb}{2V})}$$

$$C_Z = \frac{F_Z}{q_\infty S}$$

$$C_{Z_{\delta_e}} = \frac{\partial C_Z}{\partial \delta_e}$$

$$C_{Z_q} = \frac{\partial C_Z}{\partial (\frac{q\bar{c}}{2V})}$$

ACKNOWLEDGEMENTS

The author would like to express his grateful appreciation to Professor Michael H. Redlin for his invaluable guidance and counsel during this research; to Professor L. V. Schmidt for his valuable time in reading and his constructive comments on preparing this draft; to W. D. Adams, Jr., NASA Langley, for making his research available to the Aeronautics Department; and to D. S. Hague, Aerophysics Research Corporation, for his optimization program AESOP and personal interview.

I. INTRODUCTION

A. BACKGROUND

As a result of post-stall/spin accidents a significant number of lost fighter aircraft and pilot fatalities have been recorded over the last decade. Investigations have shown that these aircraft exhibit poor spin characteristics and that recovery from a fully-developed spin is usually difficult or impossible. For these reasons the spin is no longer considered a tactical maneuver and is regarded by pilots as an undesirable and potentially dangerous flight condition.

Although the flight conditions leading to a spin can generally be avoided, the combat pilot often finds the need to utilize the edge of the flight envelope in a combat situation. More often than not, it is only in this boundary area that the combat superiority of one aircraft over another is realized. If the aircraft demonstrates unsatisfactory stall and spin characteristics, the pilot is disinclined or restricted from utilizing this outermost region of flight to his best advantage. As a result, the overall weapon system effectiveness and margin of superiority of the aircraft is lost.

In light of the poor stall and spin characteristics of current fighter configurations a more detailed consideration must be given to these characteristics during the

early design stages of future fighter aircraft. In recognition of this fact this research was undertaken to provide a more accurate analytic tool with which the designer can investigate the spin characteristics of future aircraft and to aid in understanding how various factors in design may affect these spin motions. It was not proposed that this effort solve the spin problem itself, but it was hoped that it would prove to be a useful approach and that it would provide information complementary to that obtained experimentally.

B. SCOPE

Traditionally, analytic spin prediction has involved either an approximated solution (by requiring that only part of the equilibrium conditions be satisfied) or by computer generation of time-histories of characteristic parameters (by integrating the equations of motion forward in time until the average values of the variables are approximately constant). More recently the idea of mathematically casting the equations of motion into a form specifically intended for spin prediction has shown some promise. Of particular note is the work carried on by Buehler (Ref. 1) and Champoux (Ref. 2). These earlier works provided much of the initial groundwork for this research.

In developing the analytic approach to the spin problem a set of objectives was established to subdivide the research problem and also to serve as guidelines to ensure

that the work proceeded in the proper direction as effectively as possible. These objectives were:

1. To suitably model the aircraft in a spin.

2. To develop the spin equations of motion.

3. To establish a suitable technique of solving the equations of motion for the equilibrium spin condition.

4. To develop a computer program that will predict aircraft spin characteristics. In addition, the program should be flexible enough to aid in the understanding of how various parameters effect the developed spin motions.

5. To demonstrate the usefulness of the method and to verify the results with other prediction schemes.

II. DEVELOPMENT OF THE AIRCRAFT EQUATIONS OF MOTION

A. COORDINATE SYSTEM

To an observer on earth, the aircraft's center of gravity in a steady spin appears to trace a descending constant radius helical path about an imaginary vertical central axis. By assuming that the rotation of the earth is negligible, an inertial reference frame can be established that is fixed to the earth's surface at the point of intersection of this imaginary spin axis.

The most convenient method of describing this motion is through the use of cylindrical coordinates. In this formulation, the central axis of the cylinder is superimposed on the imaginary aircraft spin axis as shown in Figure 1. The advantage of this choice for an inertial coordinate system is that the center of gravity of the aircraft is conveniently located in terms of an altitude coordinate (Z_o), a radial coordinate (R) and an angular position coordinate (γ).

The body axes, as shown in Figure 2, were chosen for the formulation of the equations of motion. This allows the direct use of standard wind tunnel data which was available for this research effort. Symmetry of the aircraft about the X and Z axes was assumed so that all cross-product of inertia terms, with the exception of I_{XZ}, were identically zero.

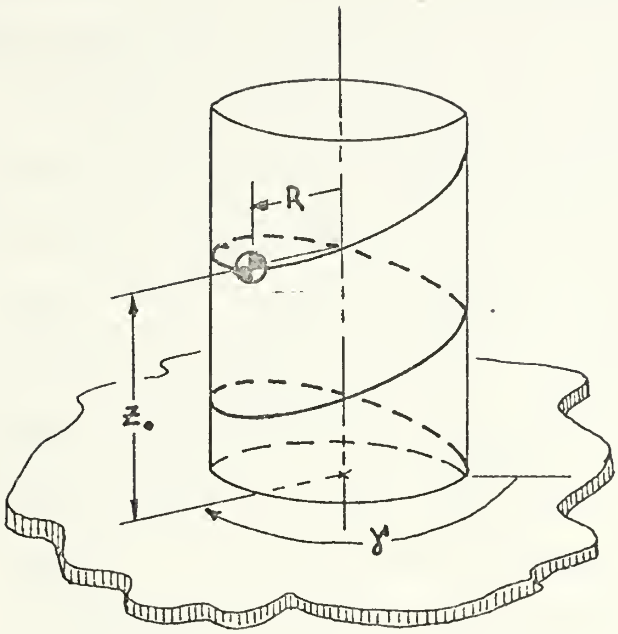

Figure 1. Inertial Cylindrical Coordinate System.

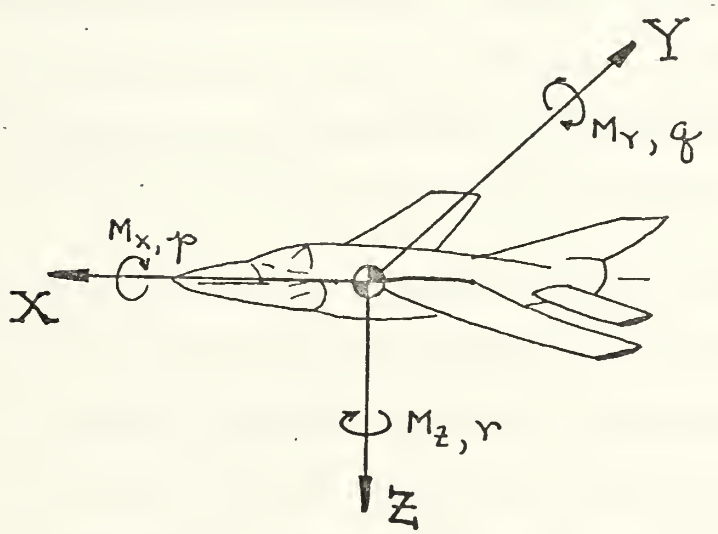

Figure 2. Aircraft Axes.

The orientation of the aircraft is described by introducing another coordinate system and a set of three rotation angles. A fixed cartesian coordinate reference frame (X_1, Y_1, Z_1) is positioned at the center of gravity of the aircraft and aligned with respect to the inertial cylindrical frame so that the Z_1 axis is parallel with the vertical central axis and the Y_1 axis coincides with the $-R$ position vector. Although this cartesian coordinate reference frame remains fixed in orientation with respect to the inertial cylindrical frame, it is still free to move in space as Z_o, R and γ take on different values. The orientation of the body axes relative to the cylindrical frame of reference is given by the aircraft Euler angles (ψ, θ, ϕ), which define an ordered series of three consecutive rotations as shown in Figure 3. The three position coordinates (Z_o, R, γ), together with the three orientation angles (ψ, θ, ϕ), allow a complete description of the motion of the aircraft in space and will serve as the generalized coordinates in the Lagrangian development of the aircraft equations of motion.

B. AXES TRANSFORMATION AND AIRCRAFT ANGULAR RATES

Before proceeding with the development of the equations of motion, certain algebraic relations are needed that will transform the actual forces, moments, velocities and angular rates of the aircraft, about its body axes, to the cylindrical reference frame. These transformations are nothing more than vector projections from one system of axes onto another.

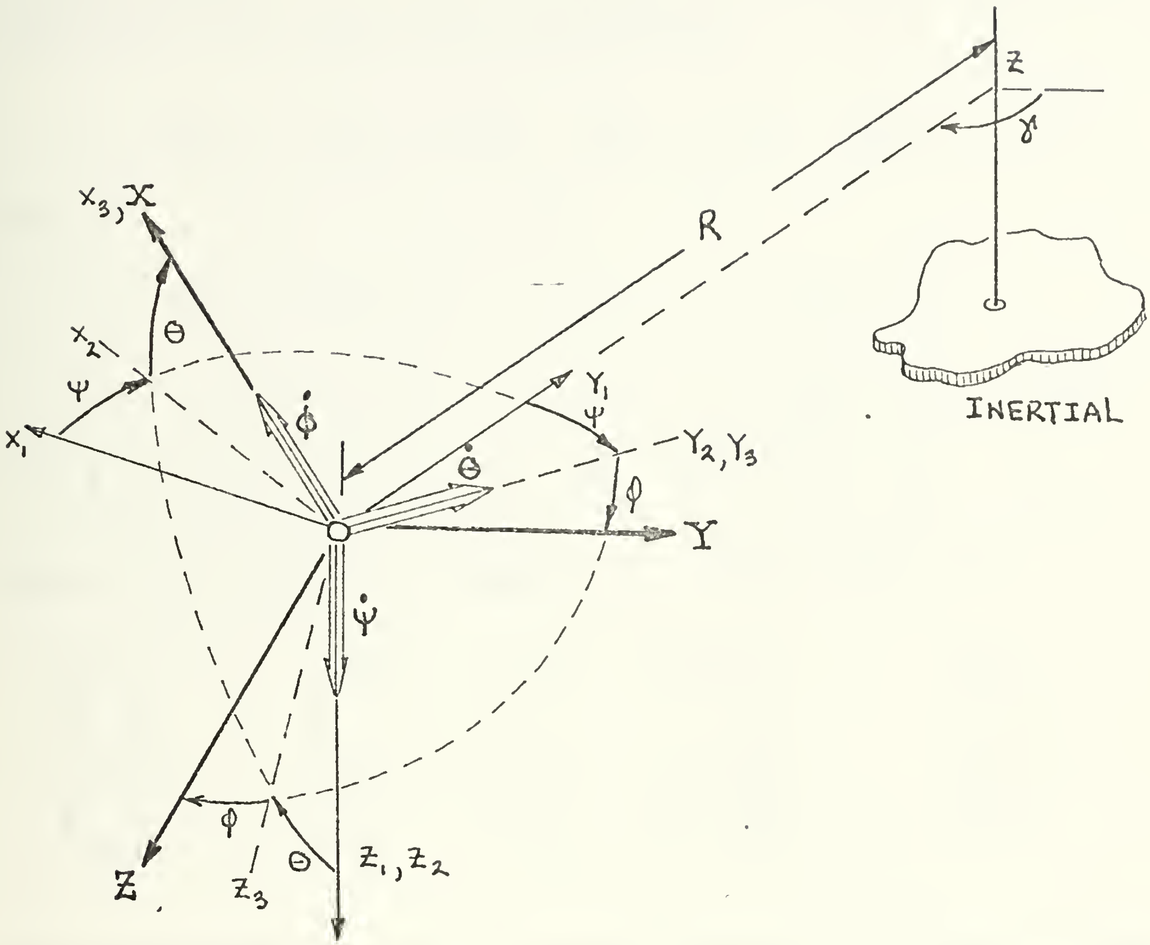

Figure 3. Spin Model.

Consider that the body axes are initially coincident with the inertial frame (X_1, Y_1, Z_1) and then are rotated about Z_1 by an angle ψ to form an intermediate reference frame (X_2, Y_2, Z_2), as shown in Figure 4. Since both of these orthogonal coordinate systems form a basis for three-space, any point (P) can be described by a vector (R) from the origin to that point in terms of

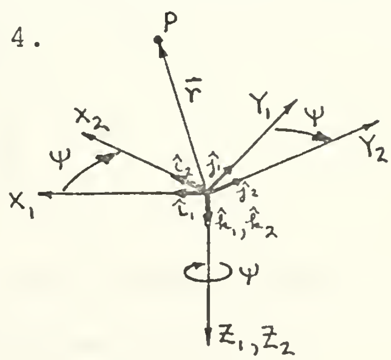

Figure 4. ψ Rotation.

either system.

Mathematically, this can be expressed:

$$\bar{r} = X_1\hat{i}_1 + Y_1\hat{j}_1 + Z_1\hat{k}_1 = X_2\hat{i}_2 + Y_2\hat{j}_2 + Z_2\hat{k}_2 \qquad (1)$$

but,

$$\begin{aligned}
\hat{i}_2 \cdot \bar{r} &= X_2 = X_1\hat{i}_2 \cdot \hat{i}_1 + Y_1\hat{i}_2 \cdot \hat{j}_1 + Z_1\hat{i}_2 \cdot \hat{k}_1 \\
\hat{j}_2 \cdot \bar{r} &= Y_2 = X_1\hat{j}_2 \cdot \hat{i}_1 + Y_1\hat{j}_2 \cdot \hat{j}_1 + Z_1\hat{j}_2 \cdot \hat{k}_1 \qquad (2) \\
\hat{k}_2 \cdot \bar{r} &= Z_2 = X_1\hat{k}_2 \cdot \hat{i}_1 + Y_1\hat{k}_2 \cdot \hat{j}_1 + Z_1\hat{k}_2 \cdot \hat{k}_1
\end{aligned}$$

which is written more compactly in matrix form as:

$$\begin{Bmatrix} X_2 \\ Y_2 \\ Z_2 \end{Bmatrix} = \begin{bmatrix} \hat{i}_2\cdot\hat{i}_1 & \hat{i}_2\cdot\hat{j}_1 & \hat{i}_2\cdot\hat{k}_1 \\ \hat{j}_2\cdot\hat{i}_1 & \hat{j}_2\cdot\hat{j}_1 & \hat{j}_2\cdot\hat{k}_1 \\ \hat{k}_2\cdot\hat{i}_1 & \hat{k}_2\cdot\hat{j}_1 & \hat{k}_2\cdot\hat{k}_1 \end{bmatrix} \begin{Bmatrix} X_1 \\ Y_1 \\ Z_1 \end{Bmatrix} \qquad (3)$$

The three-by-three matrix of unit vectors is called a transformation matrix and denoted, in this case, by $[T_{2/1}]$ and read "the transformation from frame one to two." Carrying out the indicated dot products for the rotation, the transformation matrix has the value:

$$\left[T_{2/1}\right] = \begin{bmatrix} \cos\psi & -\sin\psi & 0 \\ \sin\psi & \cos\psi & 0 \\ 0 & 0 & 1 \end{bmatrix} \qquad (4)$$

The second rotation, θ, about the Y_2 axis carries subscripted coordinate system two into system three, as

shown in Figure 3. The resulting transformation matrix is:

$$\left[T_{3/2}\right] = \begin{bmatrix} \cos\theta & 0 & \sin\theta \\ 0 & 1 & 0 \\ -\sin\theta & 0 & \cos\theta \end{bmatrix} \tag{5}$$

The final rotation, ϕ, about the X_3 axis carries subscripted coordinate system three into the unsubscripted body axes, as shown in Figure 3. This resulting transformation matrix is:

$$\left[T_{p/3}\right] = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\phi & \sin\phi \\ 0 & -\sin\phi & \cos\phi \end{bmatrix} \tag{6}$$

These individual transformation matrices can be multiplied together to form a single three-by-three matrix which will be denoted by $[T_{p/1}]$. Since each transformation matrix premultiplies the vector arrived at in the previous step, the total transformation from the cartesian coordinate frame to the aircraft body axes is given by:

$$\left[T_{p/1}\right] = \left[T_{p/3}\right] \times \left[T_{3/2}\right] \times \left[T_{2/1}\right] \tag{7}$$

Carrying out the matrix multiplication, the total transformation has the following form in which α_{ij} denotes its elements:

$$[T_{p/1}] = \begin{bmatrix} \alpha_{11} = \cos\theta\cos\psi & \alpha_{12} = \cos\theta\sin\psi & \alpha_{13} = -\sin\theta \\ \alpha_{21} = \sin\phi\sin\theta\cos\psi - \cos\phi\sin\psi & \alpha_{22} = \sin\phi\sin\theta\sin\psi + \cos\phi\cos\psi & \alpha_{23} = \sin\phi\cos\theta \\ \alpha_{31} = \cos\phi\sin\theta\cos\psi + \sin\phi\sin\psi & \alpha_{32} = \cos\phi\sin\theta\sin\psi - \sin\phi\cos\psi & \alpha_{33} = \cos\phi\cos\theta \end{bmatrix} \quad (8)$$

In relating the angular velocity rates of the aircraft, p, q and r to the inertial frame, we make use of the transformations $[T_{p/1}]$, $[T_{p/3}]$ and $[T_{3/2}]$. This development entails projecting the orientation angular rates, $\dot{\psi}$, $\dot{\theta}$ and $\dot{\phi}$ onto the aircraft body axes. In addition, the inertial angular rate ($\dot{\gamma}$) must be included to account for the rotation of the cartesian coordinate reference frame (X_1, Y_1, Z_1).

Both $\dot{\gamma}$ and $\dot{\psi}$ are considered to be applied about the Z_1 axis of the cartesian coordinate reference system and, through the $[T_{p/1}]$ transformation, their projections are realized. The Euler pitch rate ($\dot{\theta}$), about the Y_2 axis, involves only two transformations $[T_{3/2}]$ and $[T_{p/3}]$, for projection on the body axes. The Euler roll rate ($\dot{\phi}$) about the body axis involves no transformations. Mathematically expressed:

$$\begin{Bmatrix} p \\ q \\ r \end{Bmatrix} = [T_{p/1}] \begin{Bmatrix} 0 \\ 0 \\ \dot{\psi}+\dot{\gamma} \end{Bmatrix} + [T_{p/3}][T_{3/2}] \begin{Bmatrix} 0 \\ \dot{\theta} \\ 0 \end{Bmatrix} + \begin{Bmatrix} \dot{\phi} \\ 0 \\ 0 \end{Bmatrix} \quad (9)$$

On substitution of Equations (5), (6) and (8) for $[T_{3/2}]$, $[T_{p/3}]$ and $[T_{p/1}]$ respectively, in Equation (9), the exact relation between the aircraft angular rates and the Euler angle rates is given by:

$$
\begin{aligned}
p &= \dot{\phi} - \sin\theta \; (\dot{\psi}+\dot{\gamma}) \\
q &= \dot{\theta} \cos\phi + \cos\theta\sin\phi(\dot{\psi}+\dot{\gamma}) \\
r &= \cos\theta\cos\phi(\dot{\psi}+\dot{\gamma}) - \dot{\theta} \sin\phi
\end{aligned}
\tag{10}
$$

The time rates of change of the direction cosines $(\dot{\alpha}_{ij})$ and the angular rates (p,q and r) are obtained by direct differentiation and are presented in Appendix A.

C. LAGRANGE FORMULATION OF THE AIRCRAFT EQUATIONS OF MOTION

Assuming that the aircraft is a single rigid body, the three position coordinates (Z_o, R, γ) and the three orientation angles (ψ, θ, ϕ) can completely describe the six degrees of freedom of the vehicle. These six variables form the independent set of generalized coordinates necessary for Lagrange modeling of the motion. Let T denote the kinetic energy of the system; V, the system potential energy; q_r, a generalized coordinate; and F_{q_r} a generalized force due to non-conservative forces. Then the Lagrange equations for this system can be written:

$$
\frac{d}{dt}\left(\frac{\partial T}{\partial \dot{q}_r}\right) - \frac{\partial T}{\partial q_r} + \frac{\partial V}{\partial q_r} = Fq_r \; , \quad r = 1,2,\ldots 6 \tag{11}
$$

The kinetic energy of the aircraft, relative to the inertial reference frame, is given by the superposition of

the energy of the center of gravity, plus inertial rotational terms about the center of gravity, and is written:

$$T = \frac{1}{2}m[(\dot{\gamma}R)^2 + (\dot{R})^2 + (\dot{Z}_o)^2] + \frac{1}{2}[I_X p^2 + I_Y q^2 + I_Z r^2] + I_{XZ}pr \quad (12)$$

It is necessary to use the relations given in Equation (10), as Equation (12) is not compatible with the generalized coordinates $(Z_o, R, \gamma, \psi, \theta, \phi)$. Upon substitution of these relations for p, q and r the compatible kinetic energy equation is:

$$\begin{aligned} T = {} & \frac{1}{2}m[(\dot{\gamma}R)^2 + (\dot{R})^2 + (\dot{Z}_o)^2] + \frac{1}{2}I_X[\dot{\phi} - \sin\theta(\dot{\psi}+\dot{\gamma})]^2 \\ & + \frac{1}{2}I_Y[\dot{\theta}\cos\phi + \sin\phi\cos\theta(\dot{\psi}+\dot{\gamma})]^2 \qquad (13) \\ & + \frac{1}{2}I_Z[\cos\phi\cos\theta(\dot{\psi}+\dot{\gamma})]^2 \\ & + I_{XZ}[\dot{\phi} - \sin\theta(\dot{\psi}+\dot{\gamma})][(\dot{\psi}+\dot{\gamma})\cos\phi\cos\theta - \dot{\theta}\sin\phi] \end{aligned}$$

If a reference for the potential energy is established at sea level, V is given by:

$$V = mgZ_o \qquad (14)$$

The partial derivatives of Equation (13) with respect to the time rates of the generalized coordinates $(\frac{\partial T}{\partial \dot{q}_r})$ are:

$$\frac{\partial T}{\partial \dot{Z}_o} = m\dot{Z}_o \qquad (15)$$

$$\frac{\partial T}{\partial \dot{R}} = m\dot{R} \qquad (16)$$

$$\begin{aligned} \frac{\partial T}{\partial \dot{\gamma}} = {} & mR\dot{\gamma} + I_X p\alpha_{13} + I_Y q\alpha_{23} + I_Z r\alpha_{33} \\ & + I_{XZ}(p\cos\phi\cos\theta - r\sin\theta) \qquad (17) \end{aligned}$$

and

$$\frac{\partial T}{\partial \dot{\psi}} = I_X p\alpha_{13} + I_Y q\alpha_{23} + I_Z r\alpha_{33} - I_{XZ} p \sin\phi \quad (18)$$

$$\frac{\partial T}{\partial \dot{\theta}} = I_Y q \cos\phi + I_Z r \sin\phi + I_{XZ}(p\cos\phi\cos\theta - r\sin\theta) \quad (19)$$

$$\frac{\partial T}{\partial \dot{\phi}} = I_X p + I_{XZ} q \quad (20)$$

The time rates of change of Equations (15) through (20) represent $\frac{d}{dt}\left(\frac{\partial T}{\partial \dot{q}_r}\right)$ and are:

$$\frac{d}{dt}\left(\frac{\partial T}{\partial \dot{Z}_o}\right) = m\ddot{Z}_o \quad (21)$$

$$\frac{d}{dt}\left(\frac{\partial T}{\partial \dot{R}}\right) = m\ddot{R} \quad (22)$$

$$\frac{d}{dt}\left(\frac{\partial T}{\partial \dot{\gamma}}\right) = mR(R\ddot{\gamma}+2\dot{R}\dot{\gamma})+I_X(p\dot{\alpha}_{13}+\dot{p}\alpha_{13})+I_Y(q\dot{\alpha}_{23}+\dot{q}\alpha_{23}) +$$
$$I_Z(r\dot{\alpha}_{33}+\dot{r}\alpha_{33})+I_{XZ}(-p(\dot{\theta}\cos\phi\sin\theta+\dot{\phi}\sin\phi\cos\theta)+$$
$$\dot{p}\cos\phi\cos\theta - \dot{\theta}r\cos\theta - \dot{r}\sin\theta) \quad (23)$$

$$\frac{d}{dt}\left(\frac{\partial T}{\partial \dot{\psi}}\right) = I_X(p\dot{\alpha}_{13}+\dot{p}\alpha_{13})+I_Y(q\dot{\alpha}_{23}+\dot{q}\alpha_{23})+I_Z(r\dot{\alpha}_{33}+\dot{r}\alpha_{33})+$$
$$I_{XZ}(-p(\dot{\theta}\cos\phi\sin\theta+\dot{\phi}\sin\phi\cos\theta)+\dot{p}\cos\phi\cos\theta -$$
$$\dot{\theta}\, r\cos\theta - \dot{r}\sin\theta) \quad (24)$$

$$\frac{d}{dt}\left(\frac{\partial T}{\partial \dot{\theta}}\right) = I_Y(\dot{q}\cos\phi - q\dot{\phi}\sin\phi)+I_Z(\dot{r}\sin\phi+r\dot{\phi}\cos\phi)$$
$$-I_{XZ}(p\dot{\phi}\cos\phi+\dot{p}\sin\phi) \quad (25)$$

$$\frac{d}{dt}\left(\frac{\partial T}{\partial \dot{\phi}}\right) = I_X\dot{p} + I_{XZ}\dot{q} \quad (26)$$

The partial derivatives of Equation (13) with respect to the generalized coordinates $(\frac{\partial T}{\partial q_r})$ are:

$$\frac{\partial T}{\partial Z_o} = 0 \tag{27}$$

$$\frac{\partial T}{\partial R} = m\dot{\gamma}^2 R \tag{28}$$

$$\frac{\partial T}{\partial \gamma} = 0 \tag{29}$$

$$\frac{\partial T}{\partial \psi} = 0 \tag{30}$$

$$\begin{aligned}\frac{\partial T}{\partial \theta} = &-(\dot{\psi}+\dot{\gamma})(I_X p\cos\theta + I_Y q\sin\theta\sin\phi + I_Z r\sin\theta\cos\phi) \\ &+I_{XZ}(-p\cos\phi\sin\theta(\dot{\psi}+\dot{\gamma}) - r\cos\theta(\dot{\psi}+\dot{\gamma}))\end{aligned} \tag{31}$$

$$\frac{\partial T}{\partial \phi} = qr(I_Y - I_Z) + I_{XZ}(-\dot{\phi}\cos\phi - \sin\phi\cos\theta(\dot{\psi}+\dot{\gamma})) \tag{32}$$

The partial derivatives of Equation (14) with respect to the generalized coordinates $(\frac{\partial V}{\partial q_r})$ are:

$$\frac{\partial V}{\partial Z_o} = mg \tag{33}$$

$$\frac{\partial V}{\partial R} = \frac{\partial V}{\partial \gamma} = \frac{\partial V}{\partial \psi} = \frac{\partial V}{\partial \theta} = \frac{\partial V}{\partial \phi} = 0 \tag{34}$$

Equations (21) through (34) complete the formulation of the left hand sides of the six Lagrange equations, as generally expressed by Equation (11). In completing these equations it is only necessary to formulate the generalized forces F_{Z_o}, F_R, F_γ, F_ψ, F_θ and F_ϕ. The principle of virtual work forms the basis for these calculations and can be described mathematically by:

$$F_X\left(\frac{\partial X}{\partial q_r}\right) + F_Y\left(\frac{\partial Y}{\partial q_r}\right) + F_Z\left(\frac{\partial Z}{\partial q_r}\right) + M_X\left(\frac{\partial \varepsilon_X}{\partial q_r}\right) +$$

$$M_Y\left(\frac{\partial \varepsilon_Y}{\partial q_r}\right) + M_Z\left(\frac{\partial \varepsilon_Z}{\partial q_r}\right) = Fq_r \tag{35}$$

where X,Y,Z are positive incremental displacements along the X_1, Y_1, Z_1 axes, $\varepsilon_x, \varepsilon_y, \varepsilon_z$ are positive incremental angular displacements about the X_1, Y_1, Z_1 axes.

The incremental translation or rotation along or about the X_1, Y_1 or Z_1 axis, caused by varying each generalized coordinate individually, can easily be visualized with the aid of Figure 3. For example, varying the generalized coordinate Z_o by a positive increment will produce no translation or rotation about either X_1 or Y_1, and no rotation about Z_1. Using Equation (35) as a guide (realizing that each X,Y and Z appearing should be subscripted with a one, because we are dealing with the cartesian coordinate reference frame), it follows that $\frac{\partial X_1}{\partial Z_o}$, $\frac{\partial Y_1}{\partial Z_o}$, $\frac{\partial \varepsilon_X}{\partial Z_o}$, $\frac{\partial \varepsilon_Y}{\partial Z_o}$ and $\frac{\partial \varepsilon_Z}{\partial Z_o}$ are identically zero. From Figure 5 it is clear that

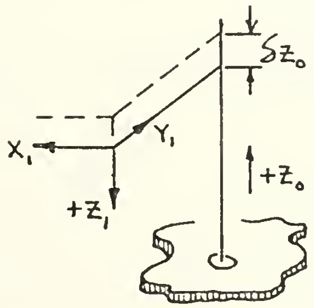

Figure 5. Positive Incremental Displacement of the Generalized Coordinate Z_o.

as Z_o is incremented a positive amount, Z_1 follows directly in its negative defined direction. This implies that $\frac{\partial Z_1}{\partial Z_o} = -1$ and the generalized force $F_{Z_o} = -F_{Z_1}$ follows from Equation (35).

The remaining generalized forces with respect to the cartesian coordinate reference frame (X_1, Y_1, Z_1) can be derived in a similar manner. The complete set is:

$$F_\psi = M_{Z_1} \tag{36}$$

$$F_\theta = -M_{X_1} \sin \psi + M_{Y_1} \cos \psi \tag{37}$$

$$F_\phi = M_{X_1} \cos\theta\cos\psi + M_{Y_1} \cos\theta\sin\psi - M_{Z_1} \sin\theta \tag{38}$$

$$F_{Z_o} = -F_{Z_1} \tag{39}$$

$$F_R = -F_{Y_1} \tag{40}$$

$$F_\gamma = F_{X_1} R + M_{Z_1} \tag{41}$$

Making use of the axes transformation $[T_{p/1}]$, the generalized forces expressed in terms of the body axes are:

$$F_R = -(\alpha_{12}F_X + \alpha_{32}F_Y + \alpha_{32}F_Z) \tag{42}$$

$$F_\gamma = -R(\alpha_{11}F_X + \alpha_{21}F_Y + \alpha_{31}F_Z) + (\alpha_{13}M_X + \alpha_{23}M_Y + \alpha_{33}M_Z) \tag{43}$$

$$F_\psi = \alpha_{13}M_X + \alpha_{23}M_Y + \alpha_{33}M_Z \tag{44}$$

$$F_\theta = M_Y \cos \phi - M_Z \sin \phi \tag{45}$$

$$F_\phi = M_X \tag{46}$$

$$F_{Z_o} = -(\alpha_{13}F_X + \alpha_{23}F_Y + \alpha_{33}F_Z) \tag{47}$$

Adding Equations (42) through (47) to the respective right-hand sides of the earlier mentioned grouping (of the left-hand terms), in the general expression given by Equation (11), completes the development. This grouping is shown in Appendix A, where the six equations of motion are presented in their entirety.

Engine thrust terms are not included in the six equations of motion. Previous studies indicate that use of thrust in the spin is normally avoided since flameouts are likely and serious damage can result (Ref. 4).

D. STEADY SPIN EQUATIONS

The full equations of motion (as given in Appendix A) can be greatly simplified for a steady state spin condition. In such a state, the aircraft is imagined to be descending at a constant velocity, at a constant radius (R) and rotation rate ($\dot{\gamma}$) about the central spin axis and with a fixed orientation with respect to the cartesian coordinate reference frame (X_1, Y_1, Z_1). This implies that all second-derivative terms and any first-derivative terms of the Euler angles (ψ, θ, ϕ) can be set equal to zero. On examination of the auxiliary terms of Appendix A, it is easily seen that all time rates of the direction cosines (α_{ij}) and angular accelerations ($\dot{p}$, $\dot{q}$, $\dot{r}$) are equal to zero. After making these simplifications, the six equations of motion that describe the steady spin condition are:

$$RES(1) = 0 = \alpha_{13}F_X + \alpha_{23}F_Y + \alpha_{33}F_Z - mg \tag{48}$$

$$RES(2) = 0 = \alpha_{12}F_X + \alpha_{22}F_Y + \alpha_{32}F_Z - m\dot{\gamma}^2 R \quad (49)$$

$$RES(3) = 0 = R(\alpha_{11}F_X+\alpha_{21}F_Y + \alpha_{31}F_Z) + (\alpha_{13}M_X + \alpha_{23}M_Y + \alpha_{33}M_Z) \quad (50)$$

$$RES(4) = 0 = \alpha_{13}M_X + \alpha_{23}M_Y + \alpha_{33}M_Z) \quad (51)$$

$$RES(5) = 0 = -\dot{\gamma}(I_X p\cos\theta + I_Y q\sin\phi\sin\theta + I_Z(r\cos\phi\sin\theta) + M_Y\cos\phi - M_Z\sin\phi - \dot{\gamma}I_{XZ}(p\cos\phi\sin\theta + r\cos\theta) \quad (52)$$

$$RES(6) = 0 = M_X - qr(I_Z - I_Y) - I_{XZ}p\dot{\gamma}\sin\phi\cos\theta \quad (53)$$

The aerodynamic coefficients are written in a form consistent with the data obtained from Ref. 5. The aerodynamic forces and moments are written:

$$F_X = \tfrac{1}{2}\rho V^2 S(C_X + C_{X_{\delta_e}} \cdot \delta_e + C_{Xq} \cdot \bar{q}) \quad (54)$$

$$F_Y = \tfrac{1}{2}\rho V^2 S(C_Y + C_{Y_{\delta_a}} \cdot \delta_a + C_{Y_{\delta_r}} \cdot \delta_r + C_{Y_p} \cdot \bar{p} + C_{Y_r} \cdot \bar{r}) \quad (55)$$

$$F_Z = \tfrac{1}{2}\rho V^2 S(C_Z + C_{Z_{\delta_e}} \cdot \delta_e + C_{Z_q} \cdot \bar{q}) \quad (56)$$

$$M_X = \tfrac{1}{2}\rho V^2 Sb(C_\ell + C_{\ell_{\delta_r}} \cdot \delta_r + C_{\ell_p} \cdot \bar{p} + C_{\ell_r} \cdot \bar{r}) \quad (57)$$

$$M_Y = \tfrac{1}{2}\rho V^2 S\bar{c}(C_m + C_{m_{\delta_e}} \cdot \delta_e + C_{m_q} \cdot \bar{q}) \quad (58)$$

$$M_Z = \tfrac{1}{2}\rho V^2 Sb(C_n + C_{n_{\delta_a}} \cdot \delta_a + C_{n_{\delta_r}} \cdot \delta_r + C_{n_p} \cdot \bar{p} + C_{n_r} \cdot \bar{r}) \quad (59)$$

If the relations given by Equations (54) through (59) were substituted into the steady spin equations, as given by Equations (48) through (53), the six equations would then describe the resulting aircraft state in terms of conventional aerodynamic forces and moments.

The residual notation (RES(i)) introduced in the steady spin equations, as given by Equations (48) through (53), is used as a measure of how closely each associated equation is being satisfied. Ideally, each residual should be identically equal to zero but this is not possible due to truncation and round-off error within the computer at various stages of the program.

A criteria function is defined that will indicate how close the complete set of six equations is to a steady spin condition. The criteria function chosen for this investigation was the sum of the squares of the individual residuals, which is written:

$$F = RES(1)^2 + RES(2)^2 + RES(3)^2 + RES(4)^2 + RES(5)^2 + RES(6)^2 \tag{60}$$

This function will be the object of the minimization effort in the optimization procedure described in the next section.

The airplane has six degrees of freedom with second order differential equations; therefore, 12 variables are required to specify its state. For convenience the 12 variables chosen are: $\psi, \dot{\psi}, \theta, \dot{\theta}, \phi, \dot{\phi}, Z_o, \dot{Z}_o, R, \dot{R}$, γ and $\dot{\gamma}$. Making use of the fact that each equality constraint imposed upon the system will remove one variable, the total number of required variables can therefore be reduced. Some of these have been discussed in describing the steady spin; specifically, that $\dot{R} = \dot{\psi} = \dot{\theta} = \dot{\phi} = 0$. They were also used to reduce the general equations given in Appendix A to the steady spin equations given by Equations (48) through (53).

By specifying that the search be made at $\gamma = 0$ and at a specified altitude (Z_o), the total number of equality constraints is six. This leaves six variables to search over if each control surface is held in a fixed position. The chosen set of variables to be searched over in this investigation are: $\dot{\gamma}$, R, $\dot{Z}_o$, ψ, θ and ϕ. Specifying these six initial conditions, the altitude parameter (ρ, as Z_o doesn't appear explicitly in the steady spin equations), the physical parameters of the aircraft (mass, chord, span, etc.), and having a set of tabulated aerodynamic coefficients, the criteria function can be evaluated. The only thing now required is a systematic method of varying the six variables ($\dot{\gamma}$, R, $\dot{Z}_o$, ψ, θ, ϕ) to obtain the minimum value of the criteria function close to zero.

The solution of the steady spin condition involves the simultaneous solution of six highly coupled nonlinear differential equations involving aerodynamic data. There does not exist a closed form analytic solution to this set of equations and therefore a numerical method must be employed.

III. REVISED FORMULATION OF THE STEADY SPIN EQUATIONS

The steady state equations were FORTRAN coded for use in program SPIN and initial efforts at predicting the equilibrium spin condition were unsuccessful. During these efforts it was realized that the rate of convergence was extremely slow and that the solutions were not close to the states already known to exist, as shown by Adams (Ref. 7).

After some analysis it was theorized that the deletion of the acceleration terms from the derived equations of motion was the cause. The equations should have been uncoupled and solved explicitly for these acceleration terms. Then the residual terms, representing the acceleration values, would be a true measure of just how close the aircraft is to the equilibrium spin condition.

Attempts at uncoupling the full equations of motion, as given in Appendix A, showed that it would be, at most, a very time consuming and nearly impossible task as the ψ, θ and ϕ equations are so highly coupled. Instead, a search for an alternate set of moment equations was made with the decision to use the equations given by Etkin (Ref. 3). The three equations express the moments about the body axes and are:

$$M_X = I_X\dot{p} - I_{XZ}\dot{r} + q_r(I_Z - I_Y) - I_{XZ}pq \qquad (61)$$

$$M_Y = I_Y\dot{q} + rp(I_X - I_Z) + I_{XZ}(p^2 - q^2) \tag{62}$$

$$M_Z = -I_{XZ}\dot{p} + I_Z\dot{r} + pq(I_Y - I_Z) + I_{XZ}q_r \tag{63}$$

The first three equations of the original formulation express Newton's second law of motion along the radial, tangential and vertical directions through the aircraft's center of gravity. These equations can be expressed, without introducing the transformations (α_{ij}), about the cartesian coordinate reference frame as:

$$F_{X_1} = m\ddot{\gamma} \tag{64}$$

$$F_{Y_1} + m\dot{\gamma}^2 R = m\ddot{R} \tag{65}$$

$$F_{Z_1} = m\ddot{Z} \tag{66}$$

These three force equations when combined with the three moment equations, as given by Equations (61) through (63), describe the motion of and rotation about the center of gravity of the aircraft and therefore completely describe the motion of the six-degree-of-freedom vehicle.

By explicitly solving Equation (64) for the acceleration term $\ddot{\gamma}$, Equation (65) for $\ddot{R}$, Equation (66) for $\ddot{Z}$ and Equation (61) through (63) for $\dot{p}$, $\dot{q}$ and $\dot{r}$ the equations are in the form desired for the optimization search.

IV. OPTIMIZATION PROCEDURE

A. CONCEPT OF OPTIMIZATION

Using state variable notation, the equilibrium solution and optimization scheme for this particular formulation can be described mathematically as follows. If we let

$$\left\{ \dot{\underline{X}} \right\} = \left\{ \begin{array}{c} \ddot{\gamma} \\ \ddot{R} \\ \ddot{Z} \\ \dot{p} \\ \dot{q} \\ \dot{r} \end{array} \right\} \quad \text{, the acceleration terms}$$

and

$$\left\{ \underline{Y} \right\} = \left\{ \begin{array}{c} \dot{\gamma} \\ R \\ \dot{Z} \\ \psi \\ \theta \\ \phi \end{array} \right\} \quad \text{, the independent variables,}$$

therefore $\left\{ \dot{\underline{X}} \right\} = \left\{ F(\underline{Y}) \right\}$. Then the optimization scheme finds $\left\{ \underline{Y} \right\}$ such that $\left\{ \dot{X} \right\} = 0$, for equilibrium. The optimization scheme employed in this investigation uses the criteria function which is the sum of the squares of the acceleration terms (modified equation residuals), given by:

$$J(\underline{Y}) = \sum_{i=1}^{6} \dot{x}_i^{\,2} = \ddot{\gamma}^2 + \ddot{R}^2 + \ddot{Z}^2 + \dot{p}^2 + \dot{q}^2 + \dot{r}^2$$

and finds $\left\{ \underline{Y} \right\}$ such that $J(\underline{Y})$ is an absolute minimum which yields $\left\{ \dot{\underline{X}} \right\} = 0$.

B. OPTIMIZATION ALGORITHM

The optimization method used in this investigation finds the minimum value of the criteria function, which is a real-valued function of several variables. The criteria function is used as an indicator of how close the aircraft is to being in an equilibrium spin condition. The minimization process takes into account any constraints imposed upon the system, which, in this case, were imposed limits of angle of attack and sideslip from the available aerodynamic coefficients.

The optimization procedure starts with a given set of independent control parameters and varies the current values of these parameters, within the imposed boundaries, to improve the value of the criteria function. The process of parameter adjustment continues until the minimum of the criteria function is reached. Depending on the hyper-surface defined by the independent control parameters, the minimum found may be just a local minimum which would not satisfy the desired steady-spin criteria functional value (equal to zero). The absolute minimum of the hyper-surface will yield a criteria function value equal to zero and is the object of the search. The initial conditions given for the independent control parameters will greatly influence the success of finding the absolute minimum. Therefore, there is a certain trial and error process in the

search for the absolute minimum without any guarantee that one exists. This then is the main disadvantage of the optimization procedure because in spite of the energy and perseverance of the researcher he cannot, at the end, say that a steady spin condition does not exist for that aircraft if in fact he hasn't been able to locate an absolute minimum. This is a very common disadvantage shared by all optimization procedures available at this time.

The optimization algorithm used in the spin program is called EXTREM. A complete listing of this subroutine is given in Appendix B along with a functional flow chart. The user specifies a set of independent variables in name and initial value and a corresponding number of scaled increments defining the initial step sizes along each of the independent variables. The algorithm first checks that the given initial conditions do not violate any constraints imposed on the system and then proceeds to determine the main line of search in the variable space. The initial conditions form a point in this variable space and the specified initial step sizes give a second. A vector joining these two points determines the initial main line of search for the first stage, as shown in Figure 6 for a two-variable problem. Along this main line the approximate minimum point is found by extrapolating a parabola through three points determined by $\overline{X}_1 - DX$, $\overline{X}_1$ and $\overline{X}_1 + DX$ and then interpolating for the minimum ($\overline{X}_{i+1}$). A Gram-Schmidt orthogonalization process determines the secondary direction of the first

stage which is orthogonal to the main direction at $\overline{X}_{i+1}$. The approximate minimum point $\overline{X}_{i+2}$ is found along this line.

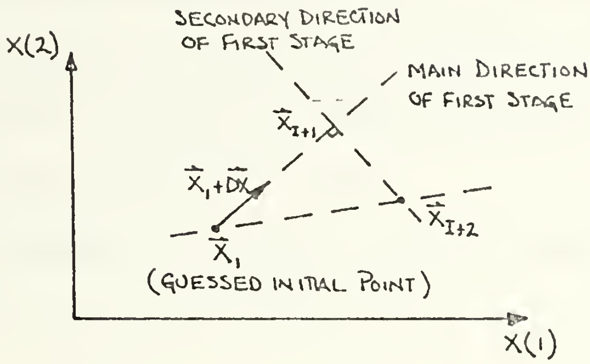

Figure 6. Main Line of Search in a Two Variable Problem

For a third variable (not shown in Figure 6) a line of search through $\overline{X}_{i+2}$ is determined by the Gram-Schmidt process that is orthogonal to both the main and secondary directions. Each new direction is orthogonal to all the preceding directions of search and the procedure for determining these lines of search and approximate minimum points is continued until the minimum along the K^{th} line (K being the number of independent variables) is found. At this point the first stage is complete and the main direction for the second stage is determined by a line joining the point $\overline{X}_1$ and $\overline{X}_{i+k}$, where $\overline{X}_{i+k}$ is the approximate minimum along the K^{th} line of the first stage. The second stage proceeds just as described for the first, and the overall stage procedure continues until a local or absolute relative minimum is found. Between stages the program checks

the stopping conditions on the criteria function value, the arguments, and the maximum number of stages allowed to reach the minimum, and will stop if any of these conditions is met. The step size along a line of search is variable in the program, which allows the search steps to decrease near the minimum or within tight curves and to increase again once these curves are past.

A more detailed description of the optimization method, the algorithm and several test cases can be found in Ref. 8.

V. DISCUSSION OF RESULTS

A. VERIFICATION EFFORT

In an attempt to verify that the spin equations of motion describe the aircraft motion and also to check out the entire computer program, several runs were made for an equilibrium flight condition of straight and level flight. The choice of this flight mode was made so that the results could be verified by hand while searching over a reduced number of independent variables in satisfying the same criteria function as discussed earlier.

As mentioned earlier, thrust terms were not included in the general equations of motion, as previous studies indicated that its use is normally avoided, since flameouts are likely and serious engine damage can result. However, in order to simulate straight and level flight thrust was included with the very restrictive assumption that it was orientated in the negative X direction and passed through the center of gravity.

Straight and level flight can be simulated in program SPIN by specifying a value of ψ equal to 90°, $\dot{\gamma}$ and $\dot{Z}$ both set to zero. By assuming that a wings-level, straight line-of-flight orientation can be obtained with neutral rudder and aileron control, the value of δ_a and δ_r are set equal to zero as well as the bank angle ϕ. The value of the

radius vector (R) is arbitrary. With this accomplished for each run a value of the pitch angle (θ) (in this special case θ is the same as the angle of attack) is specified. Three independent variables (thrust required, elevator angle and airspeed) and their initial step sizes are specified in the main program and the routine yields the value of these three variables that would be required for straight and level equilibrium flight.

The results of these runs were verified by using an electronic calculator; the equations of motion were satisfied. The numerical results of thrust, velocity and elevator deflection angle were reasonable for that particular flight configuration.

B. PROGRAM SPIN RESULTS

Several SPIN runs were conducted for configuration B in order to compare the results with those given by Adams (Ref. 9). The mass, dimensional characteristics and aerodynamic coefficients for this aircraft configuration are listed in Appendix C.

The results of the predicted spin characteristics for configuration B are shown in Table I where they are compared with the results obtained by Adams. The predicted spin characteristics developed by program SPIN, using the optimization algorithm EXTREM, are in good agreement with those predicted by Adams. The slight difference in the first, second or third decimal digit in some cases is probably a result of the particular computer being used.

Adams used a CDC machine with a bigger bit word than the IBM-360 and therefore more significant figures could be handled in the numerous calculations performed. As a result of this the minimum value of the criteria function obtained with program SPIN was 10^{-24}. Adams has been able to drive his criteria function residuals (also a sum of the squares but of a different set of terms) to a value of 10^{-36}.

The third grouping of results in Table I show very little similarity, despite the same control settings being used. The first steady spin predicted is an entirely different equilibrium condition as can be seen by comparing the values of the radius (R) and angular rate ($\dot{\gamma}$). The second spin predicted would at first appear to be the same as that given by Adams. The difference in values of the angle of attack (α), inertial velocity (V) and radius (R) make it an entirely different spin condition and not just a poor representation of the same spin. This statement is justified by the closer correlation of results, in the other groupings, for the same value of the criteria function (10^{-24}).

The computer prediction runs for configuration A are shown in Table II. There were no other results to compare with this configuration. The spin prediction scheme was applied to this particular aircraft to assure that the method would at least work for another aircraft. The choice

for this second configuration, over other possibilities, was made easy because the aerodynamic derivatives were already in tabulated form as given in Ref. 7.

The effects of density variation on the equilibrium spin conditions is shown in Table III. Grantham and Grafton (Ref. 8) concluded from their studies that a decrease in density gave a faster rotation rate ($\dot{\gamma}$), higher velocities and little change in angle of attack and sideslip. The parameters predicted by program SPIN verify this behavior with the added effect that the radius of the spin tends to decrease, i.e., the spin becomes tighter with a decrease in density. The change in the orientation angle θ would indicate that the spin would be somewhat flatter with a decrease in density.

TABLE I. Spin Characteristics for Configuration B.

(Altitude 30,000 ft)

PREDICTION METHOD	δ_e,deg	δ_a,deg	δ_r,deg	α,deg	β,deg	V ft/sec	$\dot{\gamma}$ rad/sec	ψ,deg	θ,deg	ϕ,deg	R,ft
Extrem	-25.0	7.0	-25.0	73.21	-2.381	257.9	1.1789	90.7	-16.81	-.3672	7.75
Adams*	-25.0	7.0	-25.0	73.21	-2.383	257.7	1.1781	90.7	-16.81	-.3650	7.76
Extrem	0.0	0.0	0.0	65.83	-1.959	260.8	1.1677	87.55	-24.06	.9300	10.94
Adams*	0.0	0.0	0.0	65.83	-1.960	260.5	1.1668	87.54	-24.07	.9331	10.96
Extrem	-25.0	0.0	-25.0	66.13	-1.490	272.5	0.9220	84.92	-23.60	2.02	17.32
Extrem	-25.0	0.0	-25.0	58.72	-1.333	291.6	0.7664	81.57	-30.65	4.258	33.33
Adams*	-25.0	0.0	-25.0	59.15	-1.225	290.2	0.7785	81.57	-30.24	4.228	31.89
Extrem	-25.0	7.5	-25.0	73.63	-2.41	257.3	1.2006	91.02	-16.40	-0.467	7.331
Adams*	-25.0	7.5	-25.0	73.63	-2.41	257.1	1.2001	91.02	-16.40	-0.465	7.342
Extrem	-25.0	7.0	0.0	49.18	-3.158	312.9	0.5131	77.40	-39.34	7.659	97.05
Adams*	-25.0	7.0	0.0	49.18	-3.161	312.5	0.5130	77.39	-39.33	7.665	97.03
Extrem	0.0	0.0	0.0	35.21	-9.50	323.3	1.1000	93.15	-51.71	-3.925	37.02
Adams*	0.0	0.0	0.0	35.59	-9.40	321.9	1.0952	93.09	-54.35	-3.772	36.82

*Reference 7

TABLE II. Spin Characteristics for Configuration A

(Altitude, 30,000 ft)

PREDICTION METHOD	δ_e,deg	δ_a,deg	δ_r,deg	α,deg	β,deg	V ft/sec	$\dot{\gamma}$ rad/sec	ψ,deg	θ,deg	ϕ,deg	R,ft
Extrem	-25.0	+15.0	-30.0	51.64	8.23	344.8	1.080	68.80	-36.40	15.60	23.90
Extrem	-25.0	+15.0	-30.0	48.38	10.46	341.9	0.980	69.88	-39.00	20.00	31.78
Extrem	-25.0	0.0	-30.0	39.08	2.55	340.9	0.423	61.63	-45.70	23.90	225.13
Extrem	-25.0	0.0	-30.0	50.33	8.98	390.3	1.020	71.15	-37.00	17.40	27.80
Extrem	0.0	0.0	0.0	69.34	-2.25	299.2	1.47	88.06	-20.59	-0.65	5.81
Extrem	0.0	0.0	-30.0	68.94	-2.40	300.0	1.46	90.53	-21.06	-0.77	6.05

TABLE III. Effects of Density Variation

(Configuration B)

($\delta_e = -25°$, $\delta_a = 7.5°$, $\delta_r = -25°$)

Altitude, ft	α,deg	β,deg	V ft/sec	$\dot{\gamma}$ rad/sec	ψ,deg	θ,deg	ϕ,deg	R, ft
35,000	74.1	-2.23	282.0	1.22	91.31	-15.91	-0.54	6.85
30,000	73.6	-2.40	257.4	1.20	91.02	-16.39	-0.47	7.33
25,000	73.2	-2.59	235.8	1.17	90.68	-16.86	-0.37	7.80
20,000	72.7	-2.77	216.8	1.15	90.30	-17.30	-0.26	8.29
15,000	72.3	-2.96	200.0	1.13	89.88	-17.72	-0.13	8.77
10,000	71.8	-3.15	185.1	1.11	89.41	-18.12	+0.03	9.25

VI. CONCLUSIONS AND RECOMMENDATIONS

As a result of developing and presenting the analytic spin prediction technique, the following conclusions and observations are made.

1. The original developed equations of motion, as presented in Appendix A, are too highly coupled to be an effective tool in this type of prediction scheme. In this light, those equations can be considered, at most, as an academic exercise leading to a rather unique formulation.

2. The comparison of results demonstrated the accuracy of the method and the effects of density variation study showed one possible use of the program. The capability of this method to predict steady spin conditions is limited only by the accuracy with which the mass, physical characteristics and aerodynamic coefficients are determined.

3. Due to the direct modeling in a cylindrical coordinate reference frame this method has a certain advantage over that demonstrated by Adams (Ref. 7). The number of equality constraints on the solution required by this prediction method is far fewer, which allows for a much simpler computer formulation and equally accurate results. However, this method uses more computer time than Adams' in converging to the equilibrium spin condition.

4. As with any optimization technique the importance of equally weighing the residuals of the equations is of utmost importance. The value of the residuals should be as close as possible to each other, in order of magnitude, for proper convergence of the optimization algorithm. Unequal weighting of any of these residuals cause these terms to dominate and influence the search in a direction that doesn't actually contain the desired solution minimum. Many hours of research time were lost by not recognizing this important fact.

In light of the demonstrated usefulness of the analytic prediction scheme the following follow-on work could prove

valuable in better understanding the spin problem. It is recommended that:

1. The use of rotary-balance aerodynamic data be investigated as a possible way of more closely relating the aerodynamic coefficients of the tunnel model to that experienced by the real airplane. The use of this type of data will allow at-desk calculation of the steady spin conditions. This particular spin estimation method, if used to predict the initial conditions for program SPIN, would reduce any sensitivity to the guessed initial conditions and also reduce the required computer convergence time. This particular method of spin estimation is given in Ref. 9.

2. The equations of motion could be linearized in order to obtain some information about the stability of the predicted equilibrium spin condition.

3. This prediction scheme could be coupled with existing fixed-base spin simulators to provide the starting initial conditions for the spin simulation. With this the recovery characteristics could be investigated, recovery techniques perfected and the necessary pilot interactions evaluated and studied. This one tool alone could provide valuable insight into the problem areas and shows much promise in future work.

APPENDIX A

FULL EQUATIONS OF MOTION

The following six equations completely describe the rigid body motions of an aircraft, using body axes, as formulated in a cylindrical coordinate reference frame.

Z EQUATION

$$m\ddot{Z}_o = \alpha_{13}F_X + \alpha_{23}F_Y + \alpha_{33}F_Z + mg$$

R EQUATION

$$m\ddot{R} - m\dot{\gamma}^2R = -(\alpha_{12}F_X + \alpha_{22}F_Y + \alpha_{32}F_Z)$$

γ EQUATION

$$mR(R\ddot{\gamma} + 2\dot{R}\dot{\gamma}) + I_X(p\dot{\alpha}_{13} + \dot{p}\alpha_{13}) + I_Y(q\dot{\alpha}_{23} + \dot{q}\alpha_{23})$$

$$+ I_{XZ}\{-p(\dot{\theta}\cos\phi\sin\theta + \dot{\phi}\sin\phi\cos\theta) +$$

$$\dot{p}\cos\phi\cos\theta - \dot{\theta}r\cos\theta - \dot{r}\sin\theta\} =$$

$$R(\alpha_{11}F_X + \alpha_{21}F_Y + \alpha_{31}F_Z) + \alpha_{13}M_X + \alpha_{23}M_Y + \alpha_{33}M_Z$$

ψ EQUATION

$$I_X(p\dot{\alpha}_{13} + \dot{p}\alpha_{13}) + I_Y(q\dot{\alpha}_{23} + \dot{q}\alpha_{23}) +$$

$$I_Z(r\dot{\alpha}_{33} + \dot{r}\alpha_{33}) + I_{XZ}\{-p(\dot{\theta}\cos\phi\sin\theta +$$

$$\dot{\phi}\sin\phi\cos\theta) + \dot{p}\cos\phi\cos\theta - \dot{\theta}r\cos\theta - \dot{r}\sin\theta\} =$$

$$\alpha_{13}M_X + \alpha_{23}M_Y + \alpha_{33}M_Z$$

$\underline{\theta \text{ EQUATION}}$

$$I_Y(\dot{q}\cos\phi - q\dot{\phi}\sin\phi) - I_Z(\dot{r}\sin\phi + r\dot{\phi}\cos\phi)$$
$$+ (\dot{\psi}+\dot{\gamma})\{I_X p\cos\theta + I_Y q\sin\phi\sin\theta + I_Z r\cos\phi\sin\theta\}$$
$$-I_{XZ}(\dot{\phi}\cos\phi p + \sin\phi\dot{p}) + I_{XZ}(p\cos\phi\sin\theta(\dot{\psi}+\dot{\gamma})$$
$$+ r\cos\theta(\dot{\psi}+\dot{\gamma})) = M_Y\cos\phi - M_Z\sin\phi$$

$\underline{\phi \text{ EQUATION}}$

$$I_X\dot{p} + q_r(I_Z - I_Y) - I_{XZ}(\dot{q} + p(\dot{\phi}\cos\phi + \sin\phi\cos\theta(\dot{\psi}+\dot{\gamma})))$$
$$= M_X$$

where,

$$\alpha_{11} = \cos\psi\cos\theta$$
$$\dot{\alpha}_{11} = -\dot{\psi}\sin\psi\cos\theta - \dot{\theta}\cos\psi\sin\theta$$
$$\alpha_{12} = \sin\psi\cos\theta$$
$$\dot{\alpha}_{12} = \dot{\psi}\cos\psi\cos\theta - \dot{\theta}\sin\psi\sin\theta$$
$$\alpha_{13} = -\sin\theta$$
$$\dot{\alpha}_{13} = -\dot{\theta}\cos\theta$$
$$\alpha_{21} = \cos\psi\sin\theta\sin\phi - \sin\psi\cos\phi$$
$$\dot{\alpha}_{21} = -\dot{\psi}\sin\psi\sin\theta\sin\phi - \dot{\theta}\cos\psi\cos\theta\sin\phi$$
$$-\dot{\phi}\cos\psi\sin\theta\cos\psi - \dot{\psi}\cos\psi\cos\phi + \dot{\phi}\sin\psi\sin\phi$$
$$\alpha_{22} = \sin\psi\sin\theta\sin\phi + \cos\psi\cos\phi$$
$$\dot{\alpha}_{22} = \dot{\psi}\cos\psi\sin\theta\sin\phi - \dot{\theta}\sin\psi\cos\theta\sin\phi$$
$$- \dot{\phi}\sin\psi\sin\theta\cos\phi - \dot{\psi}\sin\psi\cos\phi - \dot{\phi}\cos\psi\sin\phi$$
$$\alpha_{23} = \cos\theta\sin\phi$$
$$\dot{\alpha}_{23} = \dot{\phi}\cos\theta\cos\phi - \dot{\theta}\sin\theta\sin\phi$$
$$\alpha_{31} = \cos\psi\sin\theta\cos\phi + \sin\psi\sin\phi$$
$$\dot{\alpha}_{31} = -\dot{\psi}\sin\psi\sin\theta\cos\phi + \dot{\theta}\cos\psi\cos\theta\cos\phi$$
$$-\dot{\phi}\cos\psi\sin\theta\sin\phi + \dot{\psi}\cos\psi\sin\phi + \dot{\phi}\sin\psi\cos\phi$$

$$\alpha_{32} = \sin\psi\sin\theta\cos\phi - \cos\psi\sin\phi$$

$$\dot{\alpha}_{32} = \dot{\psi}\cos\psi\sin\theta\cos\psi + \dot{\theta}\sin\psi\cos\theta\cos\phi - \dot{\phi}\sin\phi\sin\theta\sin\phi$$
$$+ \dot{\psi}\sin\psi\sin\phi - \dot{\phi}\cos\psi\cos\phi$$

$$\alpha_{33} = \cos\theta\cos\phi$$

$$\dot{\alpha}_{33} = - \dot{\theta}\sin\theta\cos\phi - \dot{\phi}\sin\phi\cos\theta$$

$$p = \dot{\phi} - \sin\theta(\dot{\psi}+\dot{\gamma})$$

$$\dot{p} = \ddot{\phi} - \dot{\psi}\dot{\theta}\cos\theta - \ddot{\psi}\sin\theta - \dot{\gamma}\dot{\theta}\cos\theta - \ddot{\gamma}\sin\theta$$

$$q = \dot{\theta}\cos\phi + \cos\theta\sin\phi(\dot{\psi}+\dot{\gamma})$$

$$\dot{q} = - \dot{\theta}\dot{\phi}\sin\phi + \ddot{\theta}\cos\phi - \dot{\psi}\dot{\theta}\sin\theta\sin\phi - \dot{\psi}\dot{\phi}\cos\theta\cos\phi$$
$$+ \ddot{\psi}\cos\theta\sin\phi - \dot{\gamma}\dot{\theta}\sin\phi\sin\theta + \dot{\gamma}\dot{\phi}\cos\theta\cos\phi$$
$$+ \ddot{\gamma}\cos\theta\sin\phi$$

$$r = \cos\theta\ \cos\phi(\dot{\psi}+\dot{\gamma}) - \dot{\theta}\sin\phi$$

$$\dot{r} = - \dot{\psi}\dot{\theta}\sin\theta\cos\phi - \dot{\psi}\dot{\phi}\cos\theta\sin\phi + \ddot{\psi}\cos\theta\cos\phi$$
$$- \dot{\theta}\dot{\phi}\cos\phi - \ddot{\theta}\sin\phi - \dot{\gamma}\dot{\theta}\sin\theta\cos\phi$$
$$- \dot{\gamma}\dot{\phi}\cos\theta\sin\phi + \ddot{\gamma}\cos\theta\cos\phi$$

APPENDIX B

PROGRAM SPIN

A. INTRODUCTION

The intent of this appendix is to serve as a users manual or guide to the use of Program SPIN. An alphabetical listing of the computer variables and their meanings is provided. A brief description of the main purpose of each subprogram as well as an extensive explanation on preparing the data deck is included. A listing of the program with numerous comments is included along with several functional flow charts to help in understanding the structure and relationships of the subroutines. A sample output is given and the storage and time requirements of the program are discussed.

B. LIST OF COMPUTER VARIABLES

The following is an alphabetical listing of the computer variables and their meaning.

A11—A33	Direction cosines
A( )	Working matrix for Subroutine EXTREM
ALPHAD	Aircraft angle of attack, degrees
ALPHAR	Aircraft angle of attack, radians
B	Wing span, ft
BETAD	Angle of sideslip, degrees
BETAR	Angle of sideslip, radians

CBAR	Wing chord, ft
CL	Total roll moment coefficient
CM	Total pitch moment coefficient
CN	Total yaw moment coefficient
COEFA( )	Aerodynamic coefficients which are a function of angle of attack only
COEFAB( )	Aerodynamic coefficients which are a function of angle of attack and angle of sideslip
CPHI	Cosine of PHI
CPSI	Cosine of PSI
CR( )	Working vector of interpolated coefficients
CTHETA	Cosine of THETA
CX	Total X force coefficient
CY	Total Y force coefficient
CZ	Total Z force coefficient
D	1.0-(ZXI)**2/XI*ZI
DDELA	Incremental step size of aileron control, degrees
DDELE	Incremental step size of elevator control, degrees
DDELR	Incremental step size of rudder control, degrees
DELTAA	Aileron control deflection, degrees (Positive with right aileron trailing edge down)
DELTAE	Elevator control deflection, degrees(Positive with trailing edge down)
DELTAR	Rudder control deflection, degrees (Positive with trailing edge left when viewed from above)
DFMAX	Stopping condition of the functional variation in Subroutine EXTREM
DGAMD	Incremental step size of GAMDOT, rad/sec

DPHI	Incremental step size of PHI, degrees
DPSI	Incremental step size of PSI, degrees
DR	Incremental step size of R, ft
DRDOT	Incremental step size of RDOT, ft/sec
DTHETA	Incremental step size of THETA, degrees
DTHRUS	Incremental step size of THRUST, lbs
DXMAX	Control parameter for Subroutine EXTREM
DX( )	Array of step sizes of the independent variables
DZDOT	Incremental step size of ZDOT, ft/sec
F	Function to be minimized
FMAX	Desired minimum value of the optimization function
FOPT	Optimum value of the function being minimized
G	Gravitational acceleration, ft/sec2
GAMDOT	Rate of change of the cylindrical orientation variable γ, rads/sec
HRHOS	0.5*RHO*S
IW	Control parameter of Subroutine EXTREM for writing instructions +1 All output suppressed +2 Final output only +3 Output at the end of each stage
K	The number of independent variables being searched over
LMAX	Stopping condition on the number of stages of Subroutine EXTREM (A negative sign indicates that a minimum is sought)
NPRINT	Print indicator, set in MAIN and passed to Subroutine DATAIN to echo check the aerodynamic coefficients (=0, No print; =1, Prints)
P	Angular velocity about the X body axis, rad/sec
PBAR	P*B/2V

PHI	Euler orientation angle ϕ, degrees
PHIR	Euler orientation angle ϕ, radians
PSI	Euler orientation angle ψ, degrees
PSIR	Euler orientation angle ψ, radians
Q	Angular velocity about the Y body axis, rad/sec.
QBAR	Q*CBAR/2*V
R	Cylindrical radius, ft
RBAR	R*B/2*V
RDOT	Rate of change of the cylindrical radius, ft/sec.
RES( )	Residual of any of the spin equations
RHO	Density of air, lbs-sec^2/ft^4
RZ	Angular velocity about the Z body axis, rad/sec
S	Wing area, ft^2
SPHI	Sine of PHI
SPSI	Sine of PSI
SSAM	Aircraft mass, slugs
STHETA	Sine of THETA
SUMRES	Sum of the squares of the individual equation residuals
THETA	Euler orientation angle θ, degrees
THETAR	Euler orientation angle θ, radians
V	Velocity of the vehicle center of mass, ft/sec
VSQ	Velocity squared
VXB	Vehicle velocity component along the X body axis, ft/sec
VYB	Vehicle velocity component along the Y body axis, ft/sec

VYVB	VYB/V
VZB	Vehicle velocity component along the Z body axis, ft/sec
VZXB	VZB/VXB
X	Working vector that contains the independent variables
XFORCE	Aerodynamic force along the X body axis, lbs
XI	Moment of inertia about the Z body axis, slugs-ft^2
XMOM	Aerodynamic moment about the X body axis, ft-lbs
YFORCE	Aerodynamic force along the Y body axis, lbs
YI	Moment of inertia about the Y body axis, slugs-ft^2
YMOM	Aerodynamic moment about the Y body axis, ft-lbs
ZDOT	Vehicle velocity in the cylindrical axis direction, ft/sec
ZFORCE	Aerodynamic force along the Z body axis, lbs
ZI	Moment of inertia about the Z body axis, slugs-ft^2
ZMOM	Aerodynamic moment about the Z body axis, ft-lbs
ZXI	Cross product of inertia about the X and Z body axes, slugs-ft^2

C. PROGRAM DESCRIPTION

Program SPIN consists of a main controlling program and six subprograms as illustrated schematically in Figure B1.

The MAIN program's most important function is to set the independent search variables and their initial step size. Otherwise the MAIN program is basically an input/output

routine. The initial conditions and optimization control parameters are read into common memory. The initial conditions are printed out as are various final conditions of the airplane.

Subroutine DATAIN is called by the MAIN program to read in the aerodynamic coefficients. Coefficients that are a function of both alpha and beta are read into the COEFAB array and those that are a function of alpha only are read into the COEFA array. DATAIN will write out all of the coefficients if the variable NPRINT is set equal to one.

Subroutine EXTREM is the optimization program which determines the minimum of the criteria function without having to calculate any derivatives. The single most important function of this subroutine is in determining the optimum direction of search since the criteria function being minimized resides, and is evaluated, in Subroutine FN. The returned values of the criteria function are used to interpolate a parabola along a line of search, to determine the extremum on this parabola and to determine new directions of search.

Subroutine FN evaluates the criteria function value through calls to Subroutine CONST, Subroutine CLOKUP and Subroutine SSEQNS. In addition, boundaries on the search are imposed due to the limited range of angle of attack and sideslip angle aerodynamic coefficients available.

Subroutine CONST calculates various equation parameter values needed in the equations of motion. Values of

angle of attack and sideslip are determined so that the limits of the available coefficients aren't exceeded.

Subroutine CLOKUP takes the passed values of angle of attack and sideslip and performs a linear interpolation on all the aerodynamic coefficients.

Subroutine SSEQNS contains the six equations of motion of the airplane equated to the residual (RES(I), I=1,6) vector. The residuals are evaluated and passed, through common memory, to Subroutine FN so that the criteria function can be evaluated.

Functional flow charts of the MAIN program and each subroutine are shown in Figure B2 through B8.

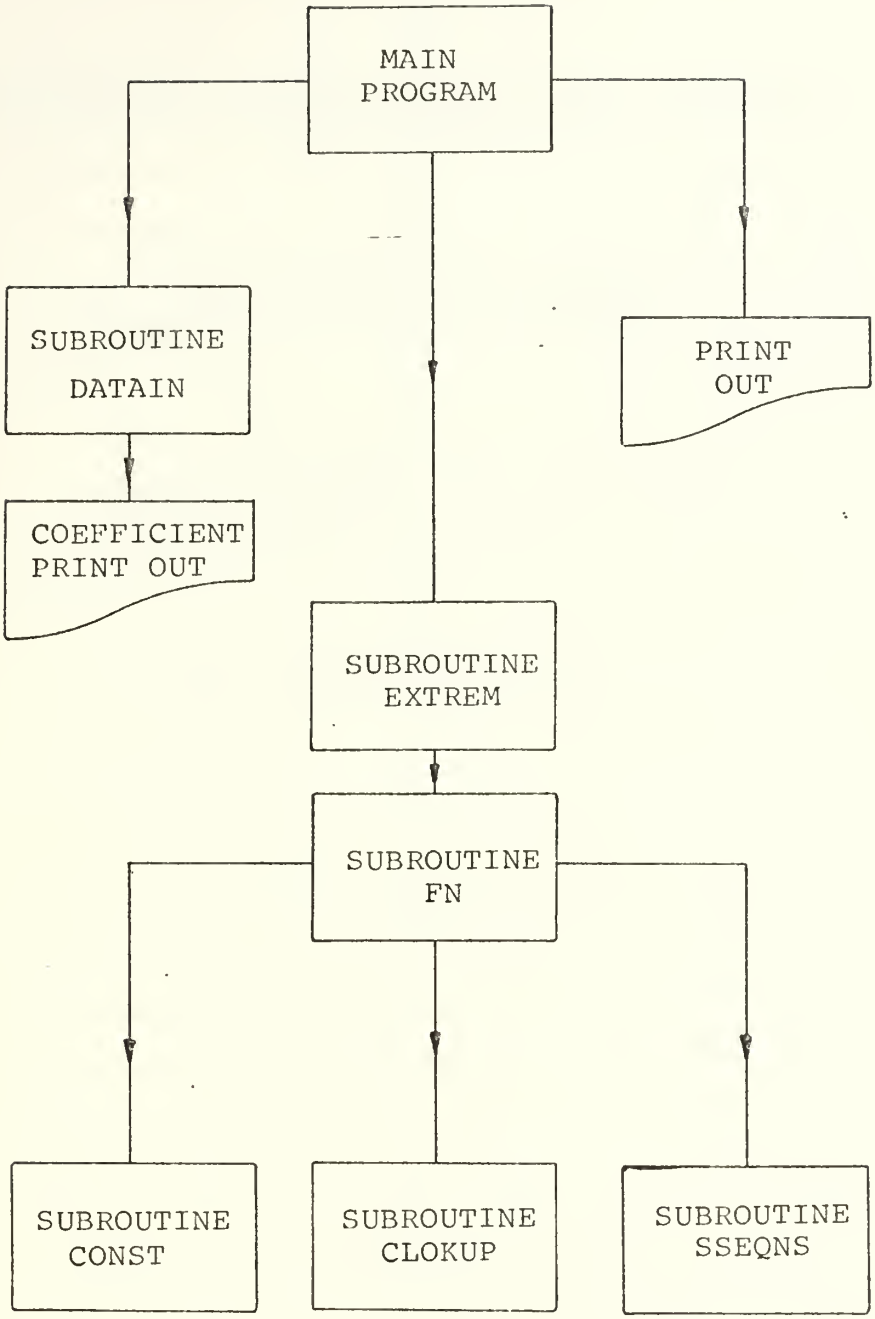

Figure B1. Schematic of Program SPIN

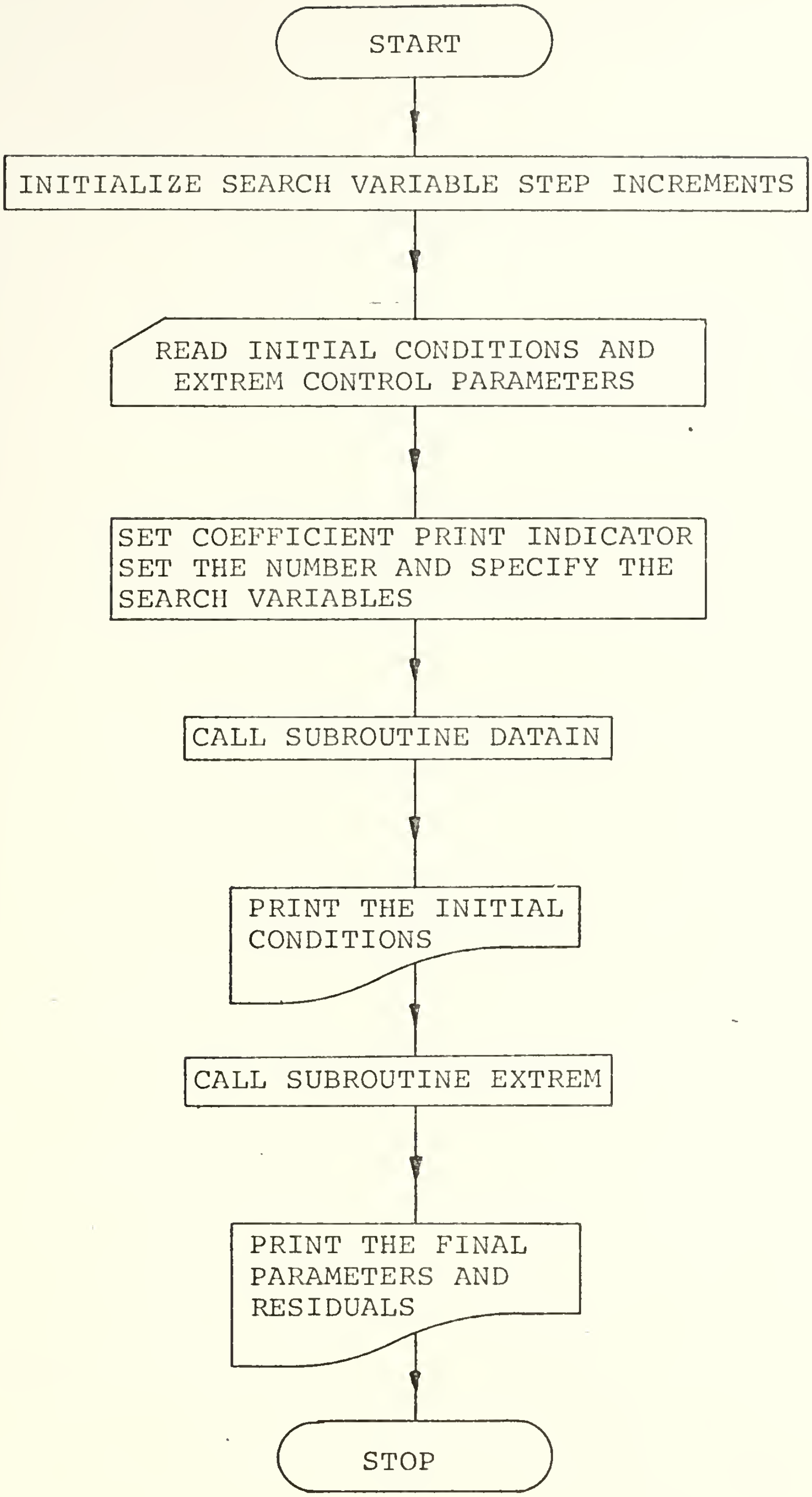

Figure B2. MAIN Program Functional Flow Chart.

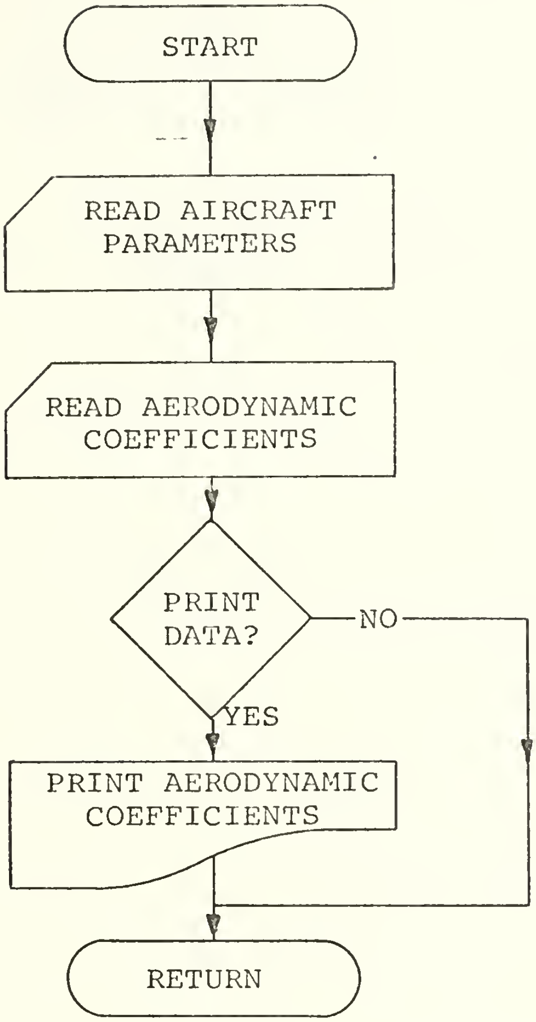

Figure B3. Subroutine DATAIN Function Flow Chart.

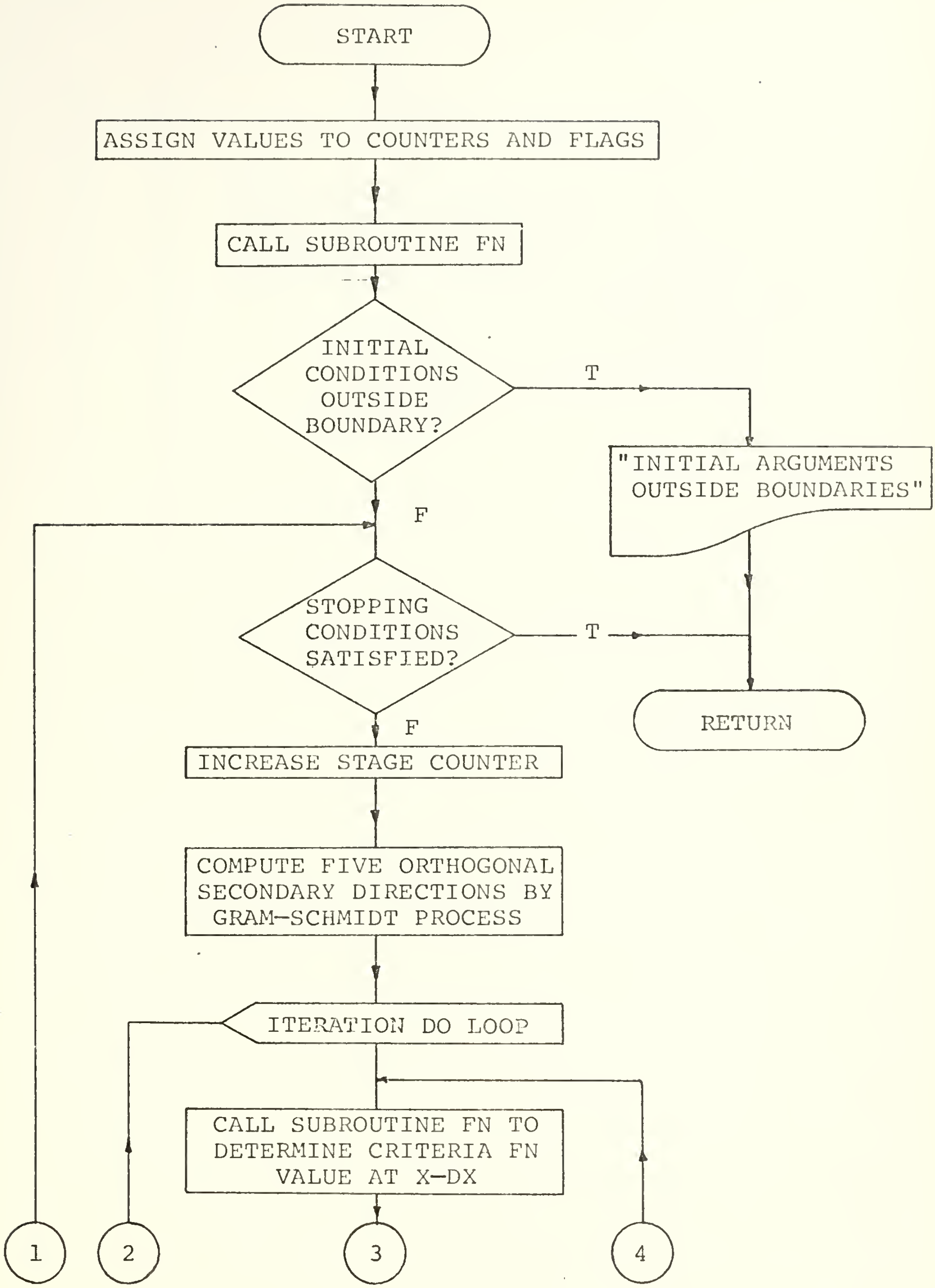

Figure B4. Subroutine EXTREM Functional Flow Chart.

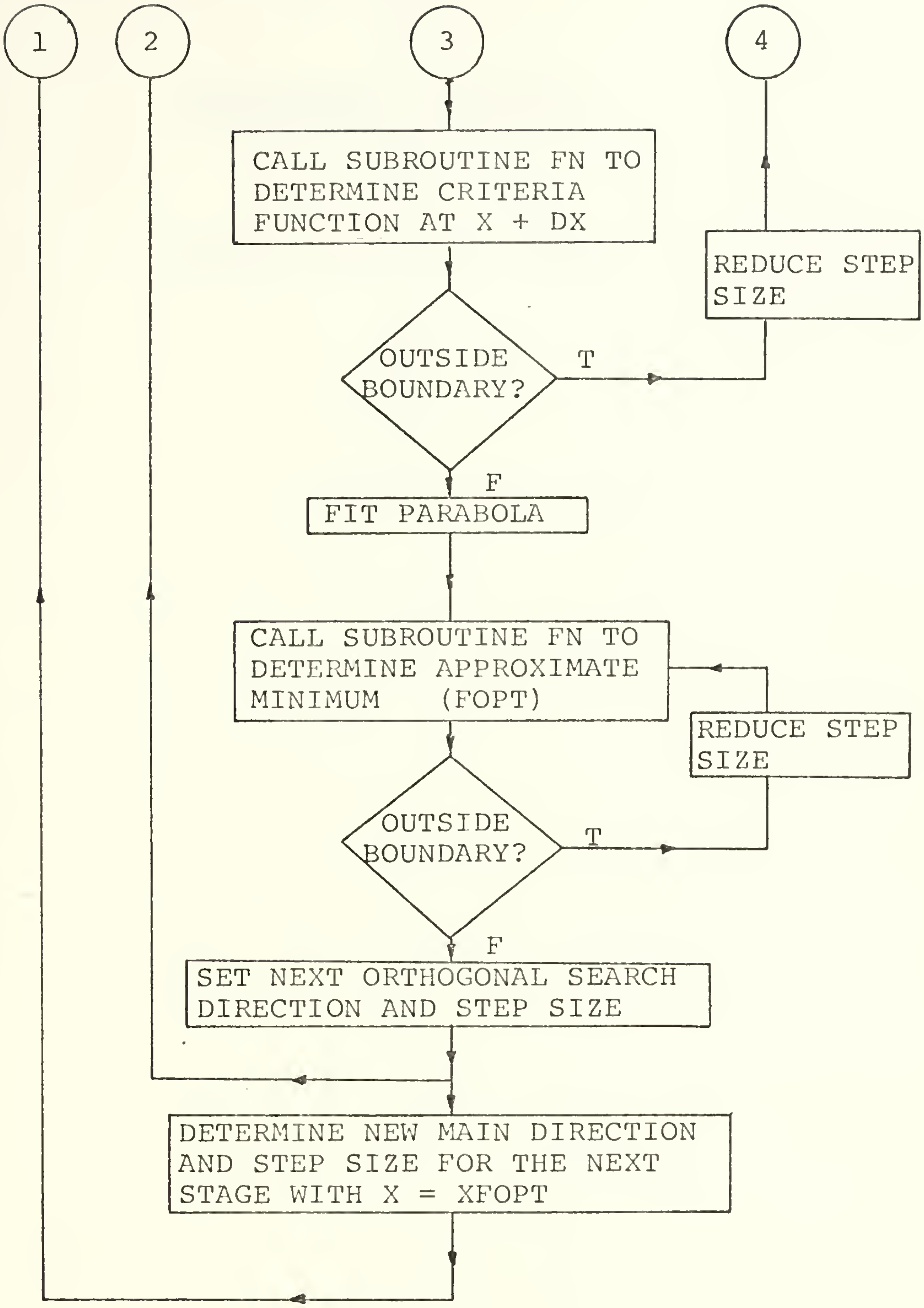

Figure B4. (Continued)

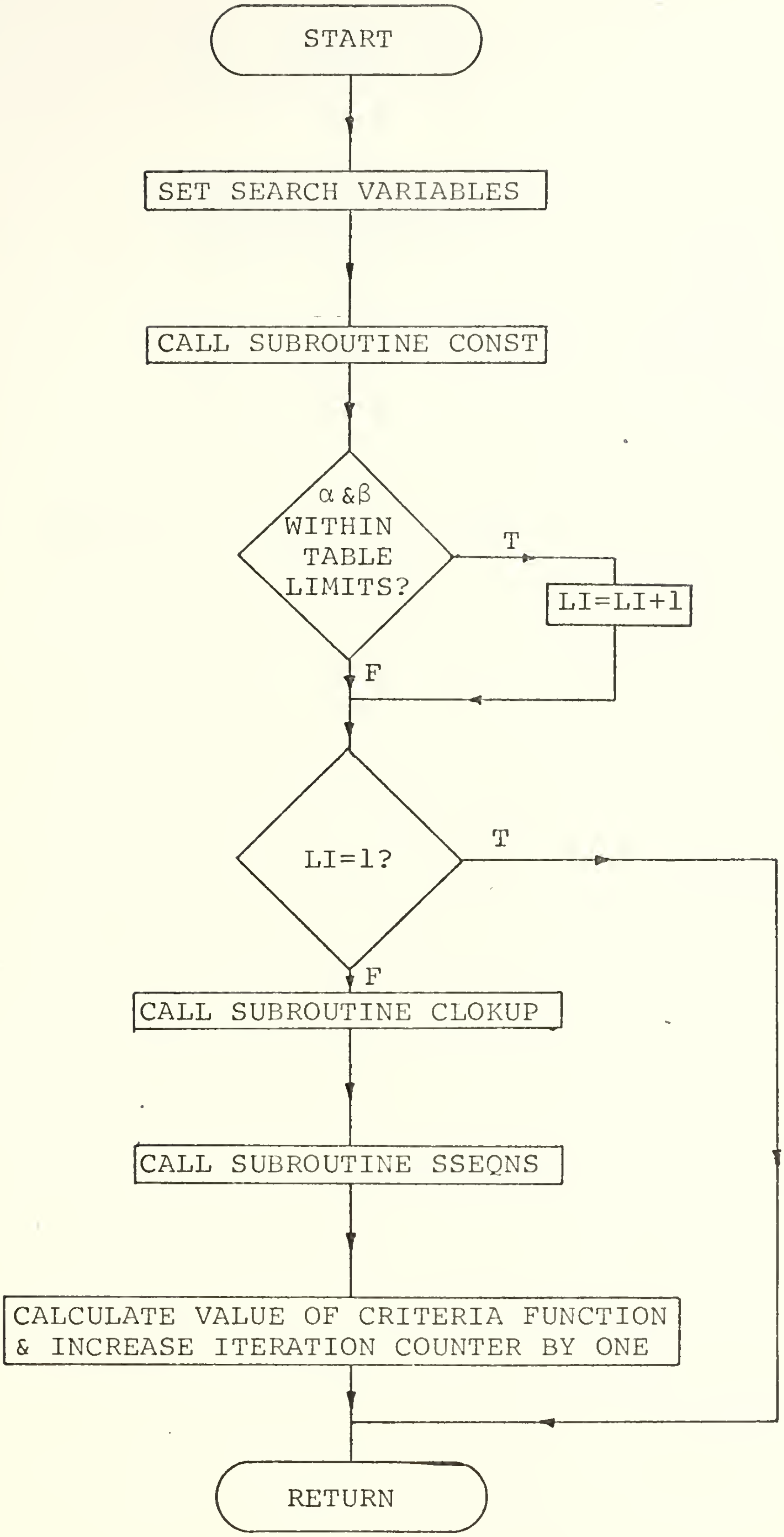

Figure B5. Subroutine FN Functional Flow Chart.

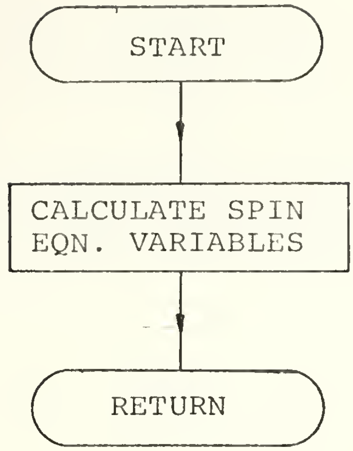

Figure B6. Subroutine CLOKUP Functional Flow Chart.

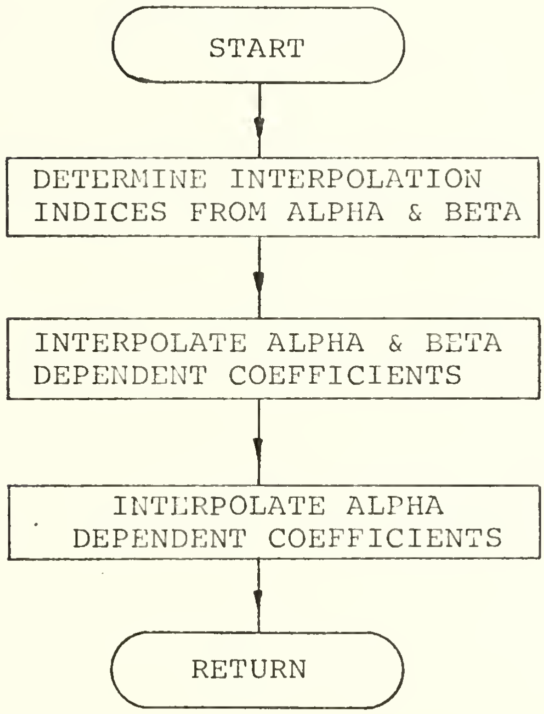

Figure B7. Subroutine CLOKUP Functional Flow Chart.

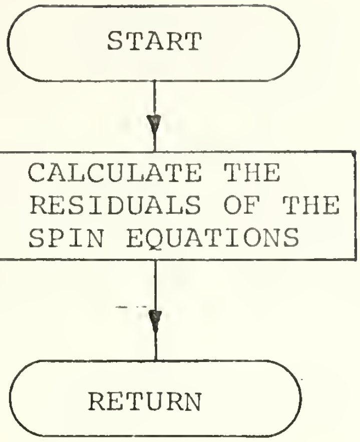

Figure B8. Subroutine SSEQNS Functional Flow Chart.

D. PREPARATION OF THE DATA DECK

The following is a guide for preparation of the data deck used by computer program SPIN.

<u>CARDS 1 and 2</u> – The initial conditions on GAMDOT (rad/sec), R(ft), RDOT(ft/sec), PHI(deg), THETA(deg), PSI(deg), and DELTAA(deg) on card one and DELTAR(deg), DELTAE(deg), THRUST(lbs) and ZDOT(ft/sec) on the second. Data is input as floating point according to the format (7F11.5/4F11.5).

<u>CARD 3</u> – The controlling parameters for Subroutine EXTREM. DFMAX, DXMAX, FMAX, LMAX and IW are read in according to the format (3E10.2,15,12).

<u>CARD 4</u> – Aircraft identification name (e.g., CONFIGURATION A) starting in column one.

<u>CARDS 5 and 6</u> – The mass and physical characteristics of the aircraft, atmospheric density and gravitational acceleration values. Card 5 contains: SSAM (mass, slugs),

S (wing area, ft^2), B (wing span, ft), CBAR ($\bar{c}$, ft) and G (gravitational acceleration, ft/sec^2). Card 6 contains: XI (I_x, slugs-ft^2), YI (I_y, slugs-ft^2), ZI (I_z, slugs-ft^2), ZXI (I_{xz}, slugs-ft^2) and RHO (atmospheric density, slugs/ft^3). Data on each card are input according to format (5F16.8).

<u>CARDS 7 - 412</u> - (alternating three card series). These cards contain the tabulated aerodynamic coefficients that are dependent on both angle of attack and sideslip. The first card in the series is as follows: in columns 1&2 the coefficient number (I), columns 3&4 blank, column 5 the index on beta (IB), columns 6-10 blank and the remaining 70 columns, in 10 column increments, are the coefficients indexed on alpha. The second card of the series is divided into eight 10 column increments and the third card is divided into four 10 column increments of aerodynamic coefficients indexed on alpha. Alpha is indexed 19 times for each beta index which corresponds to a range of alpha from 0 to +90 degrees, in 5 degree intervals. Beta is indexed nine times which corresponds to a range of beta from -40 to +40 degrees, in 10 degree intervals.

The first set of three cards, within a particular coefficient grouping, corresponds to an angle of sideslip equal to -40 degrees, where the first coefficient in columns 11-20 corresponds to an angle of attack equal to 0 degrees, columns 21-30 corresponds to an angle of attack equal to

5 degrees and so on in 5 degree increments of alpha until 90 degrees is reached in columns 41–50 of card three. The second set of three cards contains coefficients over the alpha range for beta equal to –30 degrees. The remaining series have coefficients for beta equal to –20°, –10°, 0°, +10°, +20°, +30° and +40°. The next grouping of 27 cards (nine three card series) contains another aerodynamic coefficient over the range of alpha and beta and so on in 27 card groupings until all 15 alpha and beta dependent coefficients have been prepared. Format for the three card series is (I2,2X,I1,5X,7F10.7/8F10.7/4F10.7).

The input sequencing of groupings is very important as the machine computation is dependent upon the assignment of a particular tier in the three-dimensional COEFAB array to the proper aerodynamic coefficient. Table B1 gives the proper assignment.

CARDS 413–429 – (alternating three card series). These cards contain the tabulated aerodynamic coefficients that are dependent on alpha only. The first card in the series is as follows: columns 1&2 the coefficient number (I), columns 3–10 blank and the remaining 70 columns, in 10 column increments, are the coefficients indexed on alpha. The second and third cards of the series are the same as described above for cards 6–411. Again alpha is indexed 19 times corresponding to a range of alpha from 0 to 90° in 5° increments. Columns 11–20 of card one corresponds to

the coefficient for alpha equal to zero degrees, columns 21–30 for alpha equal to 5°, and so on in 5° increments until alpha equal to 90° is reached in columns 41–50 of card three. Format for these three card series is (I2,8X, 7F10.7/8F10.7/4F10.7). As before, the input sequence of the coefficient groupings is very important for proper interpretation by the program. The assignments are given in Table B1.

TABLE B1. Coefficient Assignment

Coefficient	Storage Location	Coefficient Print Out	Interpolated Location
C_l	COEFAB (1,_,_)	1	CR(1)
C_m	COEFAB (2,_,_)	2	CR(2)
C_n	COEFAB (3,_,_)	3	CR(3)
C_y	COEFAB (4,_,_)	4	CR(4)
C_x	COEFAB (5,_,_)	5	CR(5)
C_z	COEFAB (6,_,_)	6	CR(6)
$C_{y\delta r}$	COEFAB (7,_,_)	7	CR(7)
$C_{l\delta r}$	COEFAB (8,_,_)	8	CR(8)
$C_{n\delta r}$	COEFAB (9,_,_)	9	CR(9)
$C_{y\delta a}$	COEFAB (10,_,_)	10	CR(10)
$C_{l\delta a}$	COEFAB (11,_,_)	11	CR(11)
$C_{n\delta a}$	COEFAB (12,_,_)	12	CR(12)
$C_{z\delta e}$	COEFAB (13,_,_)	13	CR(13)
$C_{m\delta e}$	COEFAB (14,_,_)	14	CR(14)
$C_{x\delta e}$	COEFAB (15,_,_)	15	CR(15)
C_{y_p}	COEFA (1,_)	16	CR(16)
C_{l_p}	COEFA (2,_)	17	CR(17)
C_{n_p}	COEFA (3,_)	18	CR(18)
C_{y_r}	COEFA (4,_)	19	CR(19)
C_{l_r}	COEFA (5,_)	20	CR(20)
C_{n_r}	COEFA (6,_)	21	CR(21)
C_{z_q}	COEFA (7,_)	22	CR(22)
C_{m_q}	COEFA (8,_)	23	CR(23)
C_{x_q}	COEFA (9,_)	24	CR(24)

E. PROGRAM LISTING

```
C*********************************************************************
C                                                                    *
C   MAIN PROGRAM                                                     *
C                                                                    *
C   PURPOSE     SERVES TO SET THE INDEPENDENT VARIABLES TO           *
C               BE SEARCHED OVER AND CONTROLS THE READING OF         *
C               THE INITIAL CONDITIONS AND OPTIMIZATION              *
C               CONTROL PARAMETERS.  ALL STRUCTURE FOR THE           *
C               PRINTED OUTPUT, WITH THE EXCEPTION OF THE            *
C               OPTIMIZATION RESULTS, COMES FROM THIS                *
C               PROGRAM.                                             *
C                                                                    *
C*********************************************************************
      IMPLICIT REAL*8(A-H,O-Z)
      EXTERNAL FN
      DIMENSION X(11),DX(11),A(11,14)
      COMMON/SPIN/GAMDOT,GAMMA,RDOT,PHI,THETA,PSI,DELTAA,DEL
     1TAE,DELTAR,THRUST,V,COEFAB(15,20,10),COEFA(20,9),SSAM,
     2S,B,CBAR,XI,YI,ZI,ZXI,G,RHO,A11,A12,A13,A21,A22,A23,A3
     31,A32,A33,VSQ,VX1,VY1,VZ1,P,Q,RZ,PBAR,QBAR,RBAR,HRHCS,
     4CR(24),RES(6),ALPHAD,ALPHAR,BETAD,BETAR,SBETA,CBETA,SA
     5LPHA,CALPHA,SGAMMA,CGAMMA,SPSI,CPSI,STHETA,CTHETA,SPHI
     6,CPHI,R,XFORCE,XFOR1,YFORCE,YFOR1,ZFORCE,ZFOR1,XMOM,YM
     7OM,ZMOM,ZDOT,NPRINT,NAME(4)
C
C         DATA INITIAL STEP SIZES OF THE INDEPENDENT
C         VARIABLES
C
      DATA DGAM/0.1/,DPSI/.01/,DZDOT/5.0/,DPHI/.01/,DR/5.0/,
     1DTHETA/.01/,DDELE/0.1/,DTHRUS/100.0/,DRDOT/5.0/
C
C         READ THE INITIAL CONDITIONS ON THE INDEPENDENT
C         VARIABLES AND OPTIMIZATION CONTROL PARAMETERS
C
      READ(5,100) GAMDOT,R,RDOT,PHI,THETA,PSI,DELTAA,DELTAR,
     1DELTAE,THRUST,ZDOT
      READ(5,101) DFMAX,DXMAX,FMAX,LMAX,IW
C
C         SET PRINT INDICATOR FOR THE AERODYNAMIC
C         COEFFICIENTS:          NPRINT=0, SUPPRESS PRINTING
C                                NPRINT=1, PRINT COEFFICIENTS
C
      NPRINT=0
C
C         SET THE NUMBER OF INDEPENDENT VARIABLES (K=) AND
C         ASSIGN EACH NAME TO THE X ARRAY WITH THE CORRES-
C         PONDING STEP SIZE TO THE DX ARRAY.
C
      K=6
      X(1)=GAMDOT
      X(2)=R
      X(3)=ZDOT
      X(4)=PSI
      X(5)=THETA
      X(6)=PHI
      DX(1)=DGAM
      DX(2)=DR
      DX(3)=DZDOT
      DX(4)=DPSI
      DX(5)=DTHETA
      DX(6)=DPHI
C
C         CALL TO SUBROUTINE DATAIN TO READ MASS, PHYSICAL
C         CHARACTERISTICS AND AERODYNAMIC COEFFICIENTS
C         OF THE AIRCRAFT.
C
      CALL DATAIN
C
C         WRITE OUT THE INITIAL CONDITIONS
C
      WRITE(6,110) NAME
      WRITE(6,102)
```

```
****************************************************************************
*                    MAIN PROGRAM CONTINUED                                *
****************************************************************************
      WRITE(6,103)
      WRITE(6,104) GAMDOT,R,RDOT,PHI,THETA
      WRITE(6,105)
      WRITE(6,104) PSI,DELTAA,DELTAR,DELTAE,THRUST
      WRITE(6,106)
      WRITE(6,119) ZDOT,RHO,DFMAX,DXMAX,FMAX
      WRITE(6,107)
      WRITE(6,108) LMAX,IW
      WRITE(6,109)
C
C          CALL THE OPTIMIZATION ALGORITHM TO FIND THE
C          STEADY SPIN PARAMETERS.
C
      CALL EXTREM(FN,K,X,DX,A,DFMAX,DXMAX,FMAX,LMAX,FOPT,IW)
C
C          WRITE OUT THE FINAL STEADY SPIN PARAMETERS.
C
      WRITE(6,110) NAME
      WRITE(6,111)
      WRITE(6,112)
      WRITE(6,104) ALPHAD,BETAD,GAMDOT,THETA,PHI
      WRITE(6,113)
      WRITE(6,104) PSI,GAMMA,R,THRUST,RHO
      WRITE(6,114)
      WRITE(6,104) V,VX1,VY1,VZ1,P
      WRITE(6,115)
      WRITE(6,116) Q,RZ,RDOT
      WRITE(6,117) XFORCE,YFORCE,ZFORCE,XMOM,YMOM,ZMOM
      DO 2 I=1,6
    2 WRITE(6,120) I,RES(I)
      WRITE(6,118) FOPT
      STOP
  100 FORMAT (7F11.5/4F11.5)
  101 FORMAT (3E10.2,I5,I2)
  102 FORMAT (T67,'- - - INITIAL CONDITIONS - - -')
  103 FORMAT (//,T58,'GAMDOT',T71,'R',T80,'RDOT',T91,
     1'PHI',T102,'THETA',/T56,'(RAD/SEC)',T69,'(DEG)',T78,
     2'(FT/SEC)',T90,'(DEG)',T102,'(DEG)',/)
  104 FORMAT (T54,5F11.5)
  105 FORMAT (//,T58,'PSI',T69,'DELTAA',T80,'DELTAR',T91,
     1'DELTAE',T102,'THRUST',/T57,'(DEG)',T69,'(DEG)',T80,
     2'(DEG)',T91,'(DEG)',T102,'(LBS)',/)
  106 FORMAT (//,T58,'ZDOT',T69,'RHO',T80,'DFMAX',T91,
     1'DXMAX',T102,'FMAX',/T55,'(FT/SEC)',/)
  107 FORMAT (//,T58,'LMAX',T69,'IW',/)
  108 FORMAT (T57,I5,T68,I3,/)
  109 FORMAT (5(/),T67,'- - - EXTREM ITERATIONS - - -',//)
  110 FORMAT ('1',8(/),T73,4A4,//)
  111 FORMAT (T67,'- - - FINAL PARAMETERS - - -')
  112 FORMAT (//,T58,'ALPHAD',T69,'BETAD',T80,'GAMDOT',T91,
     1'THETA',T102,'PHI',/T58,'(DEG)',T69,'(DEG)',T78,
     2'(RAD/SEC)',T91,'(DEG)',T101,'(DEG)',/)
  113 FORMAT (//,T59,'PSI',T69,'GAMMA',T81,'R',T90,'THRUST',
     1T103,'RHO',/T58,'(DEG)',T69,'(DEG)',T80,'(FT)',T91,
     2'(LBS)',/)
  114 FORMAT (//,T61,'V',T70,'VX1',T81,'VY1',T91,'VZ1',T105,
     1'P',/T58,'(FT/SEC)',T68,'(FT/SEC)',T79,'(FT/SEC)',T89,
     2'(FT/SEC)',T101,'(RAD/SEC)',/)
  115 FORMAT (//,T61,'Q',T70,'RZ',T81,'RDOT',/T57,'(RAD/SEC)
     1',T67,'(RAD/SEC)',T79,'(FT/SEC)',/)
  116 FORMAT(T54,3F11.5,5(/))
  117 FORMAT (///,T69,'XFORCE=',F15.5/,T69,'YFORCE=',F15.5/,
     1T69,'ZFORCE=',F15.5/,T71,'XMOM=',F15.5/,T71,'YMOM=',
     1F15.5/,T71,'ZMOM=',F15.5,2(/))
  118 FORMAT (///,T58,'SUM OF THE SQUARES',/T58,
     1'OF THE RESIDUALS',E17.5)
  119 FORMAT (T55,2F11.5,2X,3E11.4)
  120 FORMAT (//,T69,'RES(',I1,')=',E15.5)
      END
```

```
C***********************************************************************
C                                                                      *
C   SUBROUTINE DATAIN                                                  *
C                                                                      *
C   PURPOSE     READS THE MASS AND DIMENSIONAL CHARACTERISTICS
C               OF THE AIRCRAFT AS WELL AS THE AERODYNAMIC              *
C               COEFFICIENTS.                                          *
C                                                                      *
C***********************************************************************

      SUBROUTINE DATAIN
      IMPLICIT REAL*8(A-H,O-Z)
      COMMON/SPIN/GAMDOT,GAMMA,RDOT,PHI,THETA,PSI,DELTAA,DEL
     1TAE,DELTAR,THRUST,V,COEFAB(15,20,10),COEFA(20,9),SSAM,
     2S,B,CBAR,XI,YI,ZI,ZXI,G,RHO,A11,A12,A13,A21,A22,A23,A3
     31,A32,A33,VSQ,VX1,VY1,VZ1,P,Q,RZ,PBAR,QBAR,RBAR,HRFCS,
     4CR(24),RES(6),ALPHAD,ALPHAR,BETAD,BETAR,SBETA,CBETA,SA
     5LPHA,CALPHA,SGAMMA,CGAMMA,SPSI,CPSI,STHETA,CTHETA,SPHI
     6,CPHI,R,XFORCE,XFOR1,YFORCE,YFOR1,ZFORCE,ZFOR1,XMOM,YM
     7OM,ZMOM,ZDOT,NPRINT,NAME(4)
C
C           READ IN THE CONFIGURATION NAME, MASS AND PHYSICAL
C           PARAMETERS OF THE AIRPLANE, AIR DENSITY AND
C           GRAVITATIONAL ACCLERATION VALUE.
C
      READ(5,100) NAME
      READ(5,101) SSAM,S,B,CBAR,G
      READ(5,101) XI,YI,ZI,ZXI,RHO
C
C           READ THE AERODYNAMIC COEFFICIENTS THAT ARE A
C           FUNCTION OF ANGLE-OF-ATTACK AND SIDESLIP
C
      DO 1 J=1,135
    1 READ(5,102) I,IB,(COEFAB(I,IA,IB),IA=1,19)
C
C           READ THE AERODYNAMIC COEFFICIENTS THAT ARE A
C           FUNCTION OF ANGLE-OF-ATTACK ONLY
C
      DO 2 J=1,6
    2 READ(5,103) I,(COEFA(IA,I),IA=1,19)
      DO 3 J=1,3
    3 READ(5,104) I,(COEFA(IA,I),IA=1,19)
      DO 4 I=1,15
      DO 4 J=1,10
      COEFAB(I,20,J)=0.0
    4 COEFAB(I,J,10)=0.0
      DO 5 K=1,9
    5 COEFA(20,K)=0.0
C
C           CHECK PRINT INDICATOR FOR ECHO CHECK OF DATA
C
      IF(NPRINT.EQ.0) GO TO 9
      DO 7 I=1,15
      WRITE(6,105) NAME
      WRITE(6,110)
      WRITE(6,106) I
      WRITE(6,107)
      WRITE(6,108)
      WRITE(6,109)
      N=0
      DO 6 IA=1,19
      WRITE(6,111) N,(COEFAB(I,IA,IB),IB=1,9)
    6 N=N+5
    7 CONTINUE
      N=0
      WRITE(6,105) NAME
      WRITE(6,113)
      WRITE(6,112)
      DO 8 I=1,19
      WRITE(6,111) N,(COEFA(I,J),J=1,9)
    8 N=N+5
```

```
*********************************************************************
*                 SUBROUTINE DATAIN CONTINUED                       *
*********************************************************************
    9 RETURN
  100 FORMAT (4A4)
  101 FORMAT (5F16.8)
  102 FORMAT (I2,2X,I1,5X,7F10.7/8F10.7/4F10.7)
  103 FORMAT (I2,8X,7F10.7/8F10.7/4F10.7)
  104 FORMAT (I2,1X,7F11.7/7F11.7/5F11.7)
  105 FORMAT ('1',9(/),T63,4A4)
  106 FORMAT (T63,'COEFFICIENT ',I2,//)
  107 FORMAT (T29,'BETA',T40,'-40',T49,'-30',T58,'-20',T67,
     1'-10',T77,'0',T85,'+10',T94,'+20',T103,'+30',T112,
     2'+40')
  108 FORMAT (T27,'(DEGREES)')
  109 FORMAT (T29,'ALPHA')
  110 FORMAT (/)
  111 FORMAT (T30,I2,4X,9F9.5,/)
  112 FORMAT (T29,'ALPHA',T38,'COEF 16',T47,'COEF 17',T56,
     1'COEF 18',T65,'COEF 19',T74,'COEF 20',T83,'COEF 21',
     2T92,'COEF 22',T101,'COEF 23',T110,'COEF 24',/T27,
     3'(DEGREES)',/)
  113 FORMAT (/)
      END
```

```
C***********************************************************************
C                                                                      *
C   SUBROUTINE EXTREM                                                  *
C                                                                      *
C   PURPOSE     THE OPTIMIZATION ALGORITHM CALLED FROM THE             *
C               MAIN PROGRAM THAT RETURNS THE MINIMUM FOUND            *
C               FOR A PARTICULAR STEADY SPIN SEARCH.  THE              *
C               SUBROUTINE IS PROVIDED WITH AN INITIAL LINE            *
C               OF SEARCH BY THE USER THROUGH THE CALLING              *
C               ARGUMENT LIST AND THE ALGORITHM DETERMINES             *
C               THE MINIMUM ALONG THIS LINE BY INTERPOLATING           *
C               A PARABOLA THROUGH THREE CRITERIA FUNCTION             *
C               VALUE POINTS DETERMINED BY CALLS TO SUBROUTINE
C               FN.  SECONDARY DIRECTIONS OF SEARCH ARE                *
C               DETERMINED BY A GRAM-SCHMIDT ORTHOGONALIZATION
C               PROCESS AND CONTINUES, WITH VARYING STEP               *
C               SIZES, UNTIL A MINIMUM IS FOUND.                       *
C                                                                      *
C***********************************************************************

      SUBROUTINE EXTREM(F,K,X,DX,S,DFMAX,DXMAX,FMAX,LMAX,
     1FOPT,IW)
      IMPLICIT REAL*8(A-H,O-Z)
      DIMENSION X(1),DX(1),S(11,1)
C
C           ASSIGN COUNTER AND FLAG VALUES
C
      L=0
      LI=1
      N=0
      DO 1 I=1,K
      S(I,1)=X(I)
    1 S(I,2)=X(I)-DX(I)
C
C           CALL TO SUBROUTINE FN TO CHECK IF THE INITIAL
C           CONDITIONS EXCEED THE BOUNDARIES OF ANGLE-OF-
C           ATTACK AND SIDESLIP
C
      CALL F(X,F2,LI,N)
      FE=F2
      IF(LI.GT.1)WRITE(6,2)
    2 FORMAT(1X,'INITIAL ARGUMENTS OUTSIDE BOUNDARIES')
    3 IF(KC.GE.K.OR.KC.LT.0.OR.L.EQ.0)KC=0
      KC=KC+1
      S(1,3)=0.
      DO 4  I=1,K
      S(I,4)=S(I,1)-S(I,2)
    4 S(1,3)=S(1,3)+S(I,4)**2
      S(1,3)=DSQRT(S(1,3))
      IF(IABS(IW).GE.3)WRITE(6,23)L,N,S(1,3),F2,
     1(I,X(I),I,DX(I),I=1,K)
C
C           CHECK THE STOPPING CONDITIONS
C
      IF(L.GE.IABS(LMAX).OR.S(1,3).LT.DXMAX.OR.DABS(FF-FOPT)
     1.LT.DFMAX.AND.L.GT.0.OR.(LI.GT.1.AND.L.EQ.0).OR.FOPT.
     2LE.FMAX.AND.L.GT.0)GO TO 22
      IF(K.EQ.1)GOTO9
C
C           COMPUTE THE ORTHOGONAL SECONDARY DIRECTIONS BY A
C           GRAM-SCHMIDT ORTHOGONALIZATION PROCESS
C
      DO 8  J=2,K
      KD=-2+J+KC
      IF(KD.GT.K)KD=KD-K
      S(J,3)=0.
      DO 7  I=1,K
      S(I,J+3)=0.
      IF(I.EQ.KD)S(I,J+3)=S(1,3)
      JM=J-1
      DO 6  JK=1,JM
```

```
************************************************************************
*              SUBROUTINE EXTREM CONTINUED                             *
************************************************************************
    6 S(I,J+3)=S(I,J+3)-S(KD,JK+3)*S(I,3)/S(JK,3)*S(I,JK+3)
     1/S(JK,3)
    7 S(J,3)=S(J,3)+S(I,J+3)**2
      S(J,3)=DSQRT(S(J,3))
      IF(S(J,3).LT.1.D-30)GOTO3
    8 CONTINUE
    9 DO 10  I=1,K
   10 S(I,2)=S(I,1)
      L=L+1
      FF=FOPT
C
C          STAGE DO LOOP     --
C
      DO 21 M=1,K
      DO 11 I=1,K
   11 S(I,M+3)=S(I,M+3)/S(M,3)*DX(M)
      IF(IW.GT.0)LI=3
   12 IF(IW.GT.0)LI=LI-1
      LJ=LI
      DO 13 I=1,K
      X(I)=S(I,1)-S(I,M+3)
   13 S(I,M+3)=S(I,1)-X(I)
C
C          CALL SUBROUTINE FN TO DETERMINE THE CRITERIA
C          FUNCTION VALUE AT X-DX
C
      CALL F(X,F1,LI,N)
      BO=1.
   14 DO 15 I=1,K
      X(I)=S(I,1)+S(I,M+3)/BO
   15 S(I,M+3)=X(I)-S(I,1)
      IF(DABS(BO).GT.1.1)GOTO20
C
C          CALL SUBROUTINE FN TO DETERMINE THE CRITERIA
C          FUNCTION VALUE AT X+DX
C
      CALL F(X,F3,LJ,N)
C
C          IF OUTSIDE THE BOUNDARIES GO BACK AND ADJUST
C          THE STEP SIZE
C
      IF(LI+LJ.EQ.4)GOTO12
      IF(LJ.GT.2)BO=-4.
      IF(LI.GT.2)BO=+4.
      IF(LI.GT.2.OR.LJ.GT.2)GOTO14
   16 ST=0.
C
C          INTERPOLATE A PARABOLA THROUGH THE THREE
C          FUNCTIONAL VALUE POINTS: X-DX, X, X+DX
C
      DO 18 I=1,K
      X(I)=S(I,1)
      IF(DABS(S(I,M+3)).LT.1.D-30)GOTO18
      S(I,M+3)=S(I,M+3)/LI
      IF(DABS(2.*F2-F1-F3).LT.1.D-30)GOTO18
      X(I)=S(I,1)+S(I,M+3)/DABS(F1-2.*F2+F3)*(F3-F1)/
     1ISIGN(2,LMAX)
   18 ST=ST+(X(I)-S(I,1))**2
      IF(16.*ST.LT.DX(M)**2)DX(M)=DX(M)/4
      IF(ST.LT.400.*DX(M)**2.AND.DABS(2.*F2-F1-F3).GE.1.D-30
     1)GO TO 20
      DO 19 I=1,K
      IF(DABS(S(I,M+3)).LT.1.D-30)GOTO19
C
C          INTERPOLATE FOR THE APPROXIMATE MINIMUM POINT
C
      X(I)=S(I,1)+DSIGN(S(I,M+3),(F3-F1)/S(I,M+3))*
     1ISIGN(20,LMAX)
   19 CONTINUE
```

```
************************************************************************
*              SUBROUTINE EXTREM CONTINUED                             *
************************************************************************
      DX(M)=DX(M)*2.
   20 LI=+1
      BO=-BO
      IF(DABS(BO).GT.1.1)DX(M)=DX(M)/3.
C
C          DETERMINE THE MINIMUM VALUE
C
      CALL F(X,FOPT,LI,N)
      IF(LI.GT.1)LI=10
C
C          SET THE NEXT ORTHOGONAL SEARCH DIRECTION AND
C          STEP SIZE
C
      IF(ISIGN(1,LMAX)*(FOPT-F2).LT.-DABS(FE-F2)*4..AND.LI.
     1NE.10)LI=2
      IF(LI.GT.1.AND.DABS(BO).GT.1.1)GOTO14
      IF(LI.GT.1)GOTO16
      FE=F2
      F2=FOPT
      DO 21 I=1,K
   21 S(I,1)=X(I)
      GOTO3
   22 IF(IABS(IW).EQ.2)WRITE(6,23)L,N,S(1,3),F2,
     1(I,X(I),I,DX(I),I=1,K)
   23 FORMAT(//,T57,'STAGE NO.',I5,20X,'TRIAL NO.',I6,//T57,
     1'DL=',E15.5,//T57,'FUNCTION VALUE=',E15.5,///(T57,
     2'AR(',I2,')=',E15.5,5X,'DS(',I2,')=',E15.5))
      RETURN
      END
```

```
C*********************************************************************
C                                                                    *
C   SUBROUTINE FN                                                    *
C                                                                    *
C   PURPOSE     CALLED BY SUBROUTINE EXTREM FOR DETERMINATION*
C               OF THE CRITERIA FUNCTION VALUE.  SUBROUTINE  *
C               FN MAKES CALLS TO SUBROUTINE CONST, CLOKUP,   *
C               AND SSEQNS TO DETERMINE THE CRITERIA FUNCTION*
C               VALUE.  A CHECK TO INSURE THAT THE VALUES OF *
C               ANGLE-OF-ATTACK AND SIDESLIP IS PROVIDED SO  *
C               THAT A SEARCH BEYOND THE COEFFICIENT TABLE   *
C               LIMITS IS PROHIBITED.  IN SUCH A CASE, CONTROL
C               IS RETURNED TO SUBROUTINE EXTREM SO THAT THE *
C               CORRECT STEP SIZE CAN BE ADJUSTED TO VALUES  *
C               LYING WITHIN THE TABLE.                      *
C                                                                    *
C*********************************************************************


      SUBROUTINE FN(X,F,LI,N)
      IMPLICIT REAL*8(A-H,O-Z)
      DIMENSION X(1)
      COMMON/SPIN/GAMDOT,GAMMA,RDOT,PHI,THETA,PSI,DELTAA,DEL
     1TAE,DELTAR,THRUST,V,COEFAB(15,20,10),COEFA(20,9),SSAM,
     2S,B,CBAR,XI,YI,ZI,ZXI,G,RHO,A11,A12,A13,A21,A22,A23,A3
     31,A32,A33,VSQ,VX1,VY1,VZ1,P,Q,RZ,PBAR,QBAR,RBAR,HRPCS,
     4CR(24),RES(6),ALPHAD,ALPHAR,BETAD,BETAR,SBETA,CBETA,SA
     5LPHA,CALPHA,SGAMMA,CGAMMA,SPSI,CPSI,STHETA,CTHETA,SPHI
     6,CPHI,R,XFORCE,XFOR1,YFORCE,YFOR1,ZFORCE,ZFOR1,XMOM,YM
     7OM,ZMOM,ZDOT,NPRINT,NAME(4)
      GAMDOT=X(1)
      R=X(2)
      ZDOT=X(3)
      PSI=X(4)
      THETA=X(5)
      PHI=X(6)
C
C          CHECK TO SEE IF THE BOUNDARIES ARE VIOLATED.  IF
C          SO, CONTROL IS RETURNED TO SUBROUTINE EXTREM WHERE
C          THE STEP SIZE IS ADJUSTED OR NEW ARGUMENTS
C          DETERMINED THAT DO NOT VIOLATE THE BOUNDARIES
C
      CALL CONST
      IF(ALPHAD.LT.0.0.OR.ALPHAD.GT.90..OR.BETAD.LT.-40..OR.
     1BETAD.GT.40.)LI=LI+1
      IF(LI.GT.1)RETURN
C
C          CLOKUP MAKES THE INTERPOLATED COEFFICIENT VALUES
C          AVAILABLE IN COMMON MEMORY
C
      CALL CLOKUP
C
C          SSEQNS MAKES THE RESIDUALS AVAILABLE IN COMMON
C          MEMORY
C
      CALL SSEQNS
C
C          CRITERIA FUNCTION DETERMINATION
C
      F=RES(1)*RES(1)+RES(2)*RES(2)+RES(3)*RES(3)+RES(4)*
     1RES(4)+RES(5)*RES(5)+RES(6)*RES(6)
      N=N+1
      RETURN
      END
```

```
C*****************************************************************
C                                                               *
C   SUBROUTINE CONST                                            *
C                                                               *
C   PURPOSE     DETERMINES VARIOUS EQUATION PARAMETERS NEEDED*
C               IN EVALUATING THE EQUATIONS OF MOTION.  VALUES
C               OF ANGLE-OF-ATTACK AND SIDESLIP ARE PASSED    *
C               TO SUBROUTINE FN FOR A LIMIT CHECK AND ALSO   *
C               TO SUBROUTINE CLOKUP FOR COEFFICIENT INTER-   *
C               POLATION.                                       *
C                                                               *
C*****************************************************************


      SUBROUTINE CONST
      IMPLICIT REAL*8(A-H,O-Z)
      COMMON/SPIN/GAMDOT,GAMMA,RDOT,PHI,THETA,PSI,DELTAA,DEL
     1TAE,DELTAR,THRUST,V,COEFAB(15,20,10),COEFA(20,9),SSAM,
     2S,B,CBAR,XI,YI,ZI,ZXI,G,RHO,A11,A12,A13,A21,A22,A23,A3
     31,A32,A33,VSQ,VX1,VY1,VZ1,P,Q,RZ,PBAR,QBAR,RBAR,HRHOS,
     4CR(24),RES(6),ALPHAD,ALPHAR,BETAD,BETAR,SBETA,CBETA,SA
     5LPHA,CALPHA,SGAMMA,CGAMMA,SPSI,CPSI,STHETA,CTHETA,SPHI
     6,CPHI,R,XFORCE,XFOR1,YFORCE,YFOR1,ZFORCE,ZFOR1,XMOM,YM
     7OM,ZMOM,ZDOT,NPRINT,NAME(4)
      THETAR=THETA/57.295779
      PHIR=PHI/57.295779
      PSIR=PSI/57.295779
      STHETA=DSIN(THETAR)
      CTHETA=DCOS(THETAR)
      SPHI=DSIN(PHIR)
      CPHI=DCOS(PHIR)
      SPSI=DSIN(PSIR)
      CPSI=DCOS(PSIR)
      A13=-STHETA
      A23=SPHI*CTHETA
      A33=CPHI*CTHETA
      A12=CTHETA*SPSI
      A22=SPHI*STHETA*SPSI+CPHI*CPSI
      A32=CPHI*STHETA*SPSI-SPHI*CPSI
      A11=CTHETA*CPSI
      A21=SPHI*STHETA*CPSI-CPHI*SPSI
      A31=CPHI*STHETA*CPSI+SPHI*SPSI
      HRHOS=.5*RHO*S
      VSQ=(GAMDOT*R)*(GAMDOT*R)+ZDOT*ZDOT+RDOT*RDOT
      V=DSQRT(VSQ)
      P=A13*GAMDOT
      Q=A23*GAMDOT
      RZ=A33*GAMDOT
      PBAR=0.5*P*B/V
      QBAR=0.5*Q*CBAR/V
      RBAR=0.5*RZ*B/V
      ARGUM=GAMDOT*R/V
      GAMMAR=DARCOS(ARGUM)
      GAMMA=GAMMAR*57.295779
      VXB=A11*GAMDOT*R+A12*RDOT+A13*ZDOT
      VYB=A21*GAMDOT*R+A22*RDOT+A23*ZDOT
      VZB=A31*GAMDOT*R+A32*RDOT+A33*ZDOT
      VX1=A11*VXB+A21*VYB+A31*VZB
      VY1=A12*VXB+A22*VYB+A32*VZB
      VZ1=A13*VXB+A23*VYB+A33*VZB
      VZXB=VZB/VXB
      VYVB=VYB/V
      ALPHAR=DATAN(VZXB)
      BETAR=DARSIN(VYVB)
      ALPHAD=ALPHAR*57.295779
      BETAD=BETAR*57.295779
      SALPHA=DSIN(ALPHAR)
      CALPHA=DCOS(ALPHAR)
      SBETA=DSIN(BETAR)
      CBETA=DCOS(BETAR)
      RETURN
      END
```

```
C*****************************************************************
C                                                               *
C   SUBROUTINE CLCKUP                                           *
C                                                               *
C   PURPOSE     TAKES THE PASSED VALUES OF ANGLE-OF-ATTACK      *
C               AND SIDESLIP AND PERFORMS A TWO-WAY LINEAR      *
C               INTERPOLATION OF ALL 15 COEFAB (ALPHA AND       *
C               BETA DEPENDENT COEFFICIENTS ) ARRAY DERIV-      *
C               ATIVES.  A LINEAR INTERPOLATION IS PERFORMED *
C               ON THE 9 COEFA ( ALPHA ONLY DEPENDENT COEF-     *
C               FICIENTS ) ARRAY DERIVATIVES.  THE INTERPO-     *
C               LATED COEFFICIENT VALUES ( CR(I),I=1,24 ) ARE*
C               STORED IN COMMON MEMORY FOR USE BY SUBROUTINE*
C               SSEQNS IN EVALUATING THE EQUATIONS OF MOTION.*
C                                                               *
C*****************************************************************

      SUBROUTINE CLCKUP
      IMPLICIT REAL*8(A-H,O-Z)
      COMMON/SPIN/GAMDOT,GAMMA,RDOT,PHI,THETA,PSI,DELTAA,DEL
     1TAE,DELTAR,THRUST,V,COEFAB(15,20,10),COEFA(20,9),SSAM,
     2S,B,CBAR,XI,YI,ZI,ZXI,G,RHO,A11,A12,A13,A21,A22,A23,A3
     31,A32,A33,VSQ,VX1,VY1,VZ1,P,Q,RZ,PBAR,QBAR,RBAR,HRHCS,
     4CR(24),RES(6),ALPHAD,ALPHAR,BETAD,BETAR,SBETA,CBETA,SA
     5LPHA,CALPHA,SGAMMA,CGAMMA,SPSI,CPSI,STHETA,CTHETA,SPHI
     6,CPHI,K,XFORCE,XFOR1,YFORCE,YFOR1,ZFORCE,ZFOR1,XMOM,YM
     7OM,ZMOM,ZDOT,NPRINT,NAME(4)
      AI=0.2*ALPHAD+1.0
      BI=0.1*BETAD+5.0
      IA1=AI
      IB1=BI
      IA2=IA1+1
      IB2=IB1+1
      DX=BI-IB1
      DY=AI-IA1
C
C         INTERPOLATE ANGLE-OF-ATTACK AND SIDESLIP
C         DEPENDENT COEFFICIENTS
C
      DO 1 K=1,16
      C1=COEFAB(K,IA1,IB1)+DX*(COEFAB(K,IA1,IB2)-
     1COEFAB(K,IA1,IB1))
      C2=COEFAB(K,IA2,IB1)+DX*(COEFAB(K,IA2,IB2)-
     1COEFAB(K,IA2,IB1))
    1 CR(K)=C1+DY*(C2-C1)
C
C         INTERPOLATE ANGLE-OF-ATTACK DEPENDENT COEFFICIENTS
C
      DO 2 M=1,9
      I=M+15
    2 CR(I)=COEFA(IA1,M)+DY*(COEFA(IA2,M)-COEFA(IA1,M))
      RETURN
      END
```

```
C*****************************************************************
C   SUBROUTINE SSEQNS                                           *
C                                                               *
C   PURPOSE    CALCULATES THE TOTAL FORCE AND MOMENT COEF-      *
C              FICIENTS AND THE RESIDUALS ( RES(I),I=1,6 )      *
C              OF THE SIX EQUATIONS OF MOTION.  THE RESIDUALS
C              ARE MADE AVAILABLE IN COMMON MEMORY FOR          *
C              SUBROUTINE FN TO EVALUATE THE CRITERIA           *
C              FUNCTION.                                        *
C                                                               *
C*****************************************************************

      SUBROUTINE SSEQNS
      IMPLICIT REAL*8(A-H,O-Z)
      COMMON/SPIN/GAMDOT,GAMMA,RDOT,PHI,THETA,PSI,DELTAA,DEL
     1TAE,DELTAR,THRUST,V,COEFAB(15,20,10),COEFA(20,9),SSAM,
     2S,B,CBAR,XI,YI,ZI,ZXI,G,RHO,A11,A12,A13,A21,A22,A23,A3
     31,A32,A33,VSQ,VX1,VY1,VZ1,P,Q,RZ,PBAR,QBAR,RBAR,HRHOS,
     4CR(24),RES(6),ALPHAD,ALPHAR,BETAD,BETAR,SBETA,CBETA,SA
     5LPHA,CALPHA,SGAMMA,CGAMMA,SPSI,CPSI,STHETA,CTHETA,SPHI
     6,CPHI,R,XFORCE,XFOR1,YFORCE,YFOR1,ZFORCE,ZFOR1,XMOM,YM
     7OM,ZMOM,ZDOT,NPRINT,NAME(4)
C
C         CALCULATE THE TOTAL FORCE AND MOMENT COEFFICIENTS
C
      CX=CR(5)+CR(15)*DELTAE+CR(24)*QBAR
      CY=CR(4)+CR(16)*PBAR+CR(19)*RBAR+CR(10)*DELTAA+
     1CR(7)*DELTAR
      CZ=CR(6)+CR(13)*DELTAE+CR(22)*QBAR
      CL=CR(1)+CR(11)*DELTAA+CR(8)*DELTAR+CR(17)*PBAR+
     1CR(20)*RBAR
      CM=CR(2)+CR(14)*DELTAE+CR(23)*QBAR
      CN=CR(3)+CR(12)*DELTAA+CR(9)*DELTAR+CR(18)*PBAR+
     1CR(21)*RBAR
      D=1.0-ZXI*ZXI/(XI*ZI)
C
C         CALCULATE THE AERODYNAMIC FORCES AND MOMENTS
C
      XFORCE=HRHOS*VSQ*CX+THRUST
      YFORCE=HRHOS*VSQ*CY
      ZFORCE=HRHOS*VSQ*CZ
      XMOM=HRHOS*VSQ*B*CL
      YMOM=HRHOS*VSQ*CBAR*CM
      ZMOM=HRHOS*VSQ*B*CN
      XFOR1=A11*XFORCE+A21*YFORCE+A31*ZFORCE
      YFOR1=A12*XFORCE+A22*YFORCE+A32*ZFORCE
      ZFOR1=A13*XFORCE+A23*YFORCE+A33*ZFORCE+SSAM*G
C
C         CALCULATE THE EQUATION RESIDUALS
C
      RES(1)=XFOR1/(SSAM*V)
      RES(2)=(YFOR1-SSAM*GAMDOT*GAMDOT*R)/(SSAM*V)
      RES(3)=ZFOR1/(SSAM*V)
      RES(4)=(((YI-ZI)/XI-(ZXI*ZXI)/(XI*ZI))*Q*RZ+(1.-(YI-XI
     1)/ZI)*(ZXI*P*Q/XI)+(HRHOS*VSQ*B/XI)*(CL+(ZXI*CN/ZI)))
     2/D
      RES(5)=HRHOS*VSQ*CBAR*CM/YI+(ZI-XI)*P*RZ/YI+(ZXI/YI)*
     1(RZ*RZ-P*P)
      RES(6)=((ZXI*ZXI/(XI*ZI)-(YI-XI)/ZI)*P*Q+((YI-ZI)/XI-
     11.)*ZXI*Q*RZ/ZI+(HRHOS*VSQ*B/ZI)*(ZXI*CL/XI+CN))/D
      RETURN
      END
```

F, SAMPLE OUTPUT

CONFIGURATION A

\- - - INITIAL CONDITIONS - - -

GAMDOT (RAD/SEC)	R (FT)	RDOT (FT/SEC)	PHI (DEG)	THETA (DEG)
1.01000	23.00000	0.0	5.00000	-35.00000

PSI (DEG)	DELTAA (DEG)	DELTAR (DEG)	DELTAE (DEG)	THRUST (LBS)
75.00000	15.00000	-30.00000	-25.00000	0.0

ZDOT (FT/SEC)	RHO	DFMAX	DXMAX	FMAX
340.00000	0.00089	0.1000D-60	0.1000D-60	0.1000D-60

LMAX	IW
-1050	2

\- - - EXTREM ITERATIONS - - -

STAGE NO. 656 TRIAL NO. 11830

DL= 0.0

FUNCTION VALUE= 0.25726D-23

AR(1)=	0.98060D 00	DS(1)=	0.22204D-16
AR(2)=	0.69878D 02	DS(2)=	0.54210D-21
AR(3)=	0.34345D 03	DS(3)=	0.33881D-19
AR(4)=	0.20299D 02	DS(4)=	0.14211D-15
AR(5)=	-0.39163D 02	DS(5)=	0.11369D-14
AR(6)=	0.31776D 02	DS(6)=	0.84703D-20

CONFIGURATION A

- - - FINAL PARAMETERS - - -

ALPHAD (DEG)	BETAD (DEG)	GAMDOT (RAD/SEC)	THETA (DEG)	PHI (DEG)
48.38819	10.45719	0.98060	-39.16334	20.29875

PSI (DEG)	GAMMA (DEG)	R (FT)	THRUST (LBS)	RHO
69.87808	84.81609	31.77595	0.0	0.00089

V (FT/SEC)	VX1 (FT/SEC)	VY1 (FT/SEC)	VZ1 (FT/SEC)	P (RAD/SEC)
344.86437	31.15947	-0.00000	343.45381	0.61928

Q (RAD/SEC)	RZ (RAD/SEC)	RDOT (FT/SEC)
0.26376	0.71309	0.0

```
XFORCE=    2992.65300
YFORCE=   -7896.07153
ZFORCE=  -68433.28273
  XMOM=    5437.88427
  YMOM= -127703.55131
  ZMOM=   42513.27919
```

```
RES(1)=   -0.78005D-12

RES(2)=   -0.13141D-12

RES(3)=    0.13899D-11

RES(4)=   -0.32226D-13

RES(5)=    0.38337D-13

RES(6)=    0.11154D-12
```

SUM OF THE SQUARES
OF THE RESIDUALS   0.25726D-23

G. PROGRAMMING CONSIDERATIONS

Program SPIN requires 66K bytes of storage. When buffer space is taken into account, a request for 80K bytes should be made when submitting the job for processing.

The time requirement is variable and is very dependent on the closeness of the initial conditions to the steady spin condition. Convergence has been obtained with a total CPU time of a little over one minute to as high as 15 minutes; this time includes about 25 seconds for the compile and link steps.

The optimization scheme loop time is very rapid. The time required to complete one stage is 0.2 second, which requires 18 criteria function evaluations. The criteria function evaluation occurs at the rate of one per .0111 second and this accomplishes, as Figure B1 indicates, the evaluation of the equation constants, interpolation of the 24 aerodynamic coefficients, and the evaluation of the equations of motion. By looking at the listing for each of the subroutines involved, one can better appreciate the speed with which the program operates.

The variable time requirement of the program can create a problem to the researcher. More often than not, the turn around time for any program is determined by the time and storage requested. As a general guideline to the use of program SPIN, the following is suggested:

a. Set LMAX (the maximum number of stages) to a value of 1,050.

b. Request 80K storage and a four minute run time.

c. Check results of the run. If the value of the criteria function is greater than 10^{-5}, guess another set of initial conditions and rerun. Results indicate that if the value of the criteria function hasn't dropped below the above value in the four minutes alloted that an increase of time will not yield a steady spin condition. The program in this case is converging on a local minimum which is not indicative of a steady spin. If the value of the criteria function is less than 10^{-5}, rerun the program with LMAX set at 2,500 and a requested time of nine minutes if greater accuracy is desired.

APPENDIX C

AIRCRAFT DATA

The following aircraft data was used in the study of steady spin conditions. Table C1 tabulates the mass and dimensional characteristics of the two aircraft and the following pages list the aerodynamic coefficients for each configuration.

TABLE C1. Mass and Dimensional Characteristics

Characteristics	Configuration A	Configuration B
m (slugs)	1554.00	771.81
s (ft^2)	525.00	695.00
b (ft)	63.00	38.00
$\bar{c}$ (ft)	9.04	23.76
I_x (slugs-ft^2)	53100.00	13600.00
I_y (slugs-ft^2)	299000.00	128000.00
I_z (slugs-ft^2)	338750.00	138000.00
I_{xz} (slugs-ft^2)	12480.00	4340.00

CONFIGURATION A

C_ℓ

COEFFICIENT 1

BETA (DEGREES) ALPHA	-40	-30	-20	-10	0	+10	+20	+30	+40
0	0.04364	0.03202	0.02040	0.00878	0.0	-0.01446	-0.02608	-0.03770	-0.04932
5	0.07190	0.05351	0.03513	0.01676	0.0	-0.02001	-0.03838	-0.05676	-0.07515
10	0.10015	0.07501	0.04987	0.02473	0.0	-0.02555	-0.05069	-0.07583	-0.10097
15	0.09014	0.06703	0.04392	0.02081	0.0	-0.02541	-0.04852	-0.07163	-0.09474
20	0.08013	0.05905	0.03797	0.01689	0.0	-0.02527	-0.04635	-0.06743	-0.08851
25	0.07871	0.05087	0.02480	0.00615	-0.00358	-0.02169	-0.02602	-0.05594	-0.07973
30	0.07729	0.04270	0.01162	-0.00459	-0.00716	-0.01811	-0.00568	-0.04446	-0.07094
35	0.06300	0.04297	0.01635	-0.00473	-0.01567	-0.01419	-0.01203	-0.04392	-0.06837
40	0.05243	0.04297	0.02081	-0.00513	-0.02094	-0.01054	-0.01865	-0.04351	-0.06608
45	0.07337	0.04324	0.02716	0.00595	-0.01040	-0.01013	-0.02838	-0.05324	-0.07283
50	0.07810	0.04094	0.02784	0.01216	-0.00419	-0.01122	-0.02811	-0.05270	-0.06540
55	0.08310	0.06689	0.04716	0.02054	0.00122	-0.02270	-0.04689	-0.05635	-0.07878
60	0.08283	0.06756	0.05040	0.02446	0.00257	-0.02324	-0.05108	-0.06878	-0.08202
65	0.08364	0.06675	0.05459	0.02648	0.00068	-0.02675	-0.05000	-0.06851	-0.08270
70	0.08716	0.06635	0.04702	0.02770	0.00135	-0.02554	-0.04729	-0.06500	-0.08432
75	0.08628	0.06568	0.04540	0.02695	-0.00094	-0.02439	-0.04628	-0.06568	-0.08398
80	0.08540	0.06500	0.04378	0.02621	-0.00324	-0.02324	-0.04527	-0.06635	-0.08364
85	0.08527	0.06365	0.04574	0.02655	0.00054	-0.02392	-0.04635	-0.06439	-0.08283
90	0.08513	0.06229	0.04770	0.02689	0.00432	-0.02459	-0.04743	-0.06243	-0.08202

CONFIGURATION A

C_m

COEFFICIENT 2

BETA (DEGREES) ALPHA	-40	-30	-20	-10	0	+10	+20	+30	+40
0	0.05738	0.05738	0.05738	0.05738	0.05738	0.05738	0.05738	0.05738	0.05738
5	-0.07952	-0.07952	-0.07952	-0.07952	-0.07952	-0.07952	-0.07952	-0.07952	-0.07952
10	-0.21642	-0.21642	-0.21642	-0.21642	-0.21642	-0.21642	-0.21642	-0.21642	-0.21642
15	-0.26996	-0.26996	-0.26996	-0.26996	-0.26996	-0.26996	-0.26996	-0.26996	-0.26996
20	-0.32349	-0.32349	-0.32349	-0.32349	-0.32349	-0.32349	-0.32349	-0.32349	-0.32349
25	-0.19995	-0.23973	-0.39499	-0.36781	-0.50925	-0.44762	-0.41576	-0.35934	-0.03640
30	-0.07640	-0.15597	-0.46649	-0.41212	-0.69501	-0.57175	-0.50803	-0.39519	0.25069
35	0.11954	-0.17793	-0.46170	-0.50362	-0.76088	-0.61487	-0.52656	-0.35540	0.06558
40	0.32591	-0.19988	-0.44649	-0.59171	-0.76884	-0.65458	-0.53467	-0.30159	-0.11611
45	-0.01630	-0.02026	-0.32489	-0.63015	-0.83027	-0.68553	-0.32815	-0.46259	-0.33607
50	-0.17745	0.03665	-0.34542	-0.66994	-0.87954	-0.69063	-0.36595	-0.54503	-0.21319
55	-0.31560	-0.26044	-0.75853	-0.81288	-0.93924	-0.84009	-0.42907	-0.55299	-0.48806
60	-0.68176	-0.41570	-0.39101	-1.00830	-1.05820	-0.91243	-0.60207	-0.70793	-0.54703
65	-0.76873	-0.65961	-0.51811	-1.12490	-1.12030	-0.93360	-1.01270	-0.90983	-0.62669
70	-0.69862	-1.10480	-1.21900	-1.07110	-1.23660	-1.21000	-1.40560	-0.97285	-0.73388
75	-0.83606	-1.19945	-1.40125	-1.39435	-1.51180	-1.45710	-1.48410	-1.07653	-0.84581
80	-0.97349	-1.29410	-1.58350	-1.71760	-1.78700	-1.70420	-1.56260	-1.18020	-0.95774
85	-1.09670	-1.42035	-1.67020	-1.85625	-2.02855	-1.85110	-1.67205	-1.32225	-1.01957
90	-1.21990	-1.54660	-1.75690	-1.99490	-2.27010	-1.99800	-1.78150	-1.46430	-1.08140

CONFIGURATION A

C_n

COEFFICIENT 3

BETA (DEGREES) ALPHA	-40	-30	-20	-10	0	+10	+20	+30	+40
0	-0.05437	-0.04085	-0.02733	-0.01381	0.0	0.01323	0.02675	0.04027	0.05379
5	-0.04703	-0.03548	-0.02392	-0.01235	0.0	0.01078	0.02235	0.03391	0.04547
10	-0.03972	-0.03011	-0.02050	-0.01089	0.0	0.00833	0.01794	0.02755	0.03716
15	-0.02284	-0.01763	-0.01242	-0.00720	0.0	0.00323	0.00844	0.01366	0.01888
20	-0.00597	-0.00515	-0.00433	-0.00351	0.0	-0.00187	-0.00105	-0.00023	0.00059
25	0.02202	0.01371	0.02002	0.00867	-0.00292	-0.02300	-0.02269	-0.02609	-0.02179
30	0.05000	0.03257	0.04437	0.02085	-0.00584	-0.04413	-0.04432	-0.05194	-0.04417
35	0.05265	0.03998	0.03807	0.03627	-0.00715	-0.05908	-0.04729	-0.05734	-0.06126
40	0.05487	0.04711	0.03149	0.05140	-0.01611	-0.07474	-0.05055	-0.06317	-0.07835
45	0.06047	0.03425	0.00685	0.02701	-0.02480	-0.06757	-0.00870	-0.06445	-0.08460
50	0.05398	0.01695	-0.00982	-0.02688	-0.03349	-0.02011	0.00091	-0.06191	-0.08538
55	0.03855	-0.00187	-0.03703	-0.02543	0.00789	0.02198	0.01525	-0.04962	-0.06555
60	0.04171	-0.02543	-0.05026	0.01767	0.08649	0.03504	0.01368	-0.03081	-0.05082
65	0.02957	-0.02887	-0.06256	-0.00762	0.08312	0.05892	-0.02481	-0.02822	-0.04056
70	-0.00685	-0.00649	0.01352	-0.05226	-0.04174	-0.02197	-0.07930	-0.00351	-0.02611
75	-0.01503	-0.02688	0.01328	-0.02850	-0.01574	-0.01181	-0.04533	0.01927	-0.00912
80	-0.02321	-0.04728	0.01303	-0.00475	0.01026	-0.00165	-0.01137	0.04204	0.00787
85	-0.02244	-0.02113	0.00324	-0.00303	0.00268	0.00109	0.00375	0.02513	0.01403
90	-0.02166	0.00502	-0.00655	-0.00131	-0.00490	0.00382	0.01888	0.00821	0.02018

CONFIGURATION A

C_y

COEFFICIENT 4

BETA (DEGREES) ALPHA	-40	-30	-20	-10	0	+10	+20	+30	+40
0	0.53076	0.39807	0.26538	0.13269	0.0	-0.13269	-0.26538	-0.39807	-0.53076
5	0.54795	0.41281	0.27741	0.14252	0.0	-0.12778	-0.26292	-0.39807	-0.51971
10	0.56514	0.42754	0.28994	0.15234	0.0	-0.12286	-0.26046	-0.39806	-0.53566
15	0.54549	0.41526	0.28478	0.15480	0.0	-0.10566	-0.23589	-0.36612	-0.49635
20	0.52584	0.40298	0.28012	0.15726	0.0	-0.08846	-0.21132	-0.33418	-0.45704
25	0.52830	0.39561	0.27275	0.13515	0.02703	-0.05651	-0.17692	-0.31452	-0.43247
30	0.53075	0.38823	0.26537	0.11303	0.05406	-0.02457	-0.14252	-0.29486	-0.40789
35	0.56023	0.41280	0.30469	0.17200	0.04914	-0.05897	-0.19166	-0.32926	-0.42755
40	0.59463	0.43738	0.34400	0.23097	0.04914	-0.08846	-0.24080	-0.35875	-0.44720
45	0.57006	0.53566	0.39806	0.25555	0.03931	-0.12777	-0.28995	-0.43738	-0.47178
50	0.57498	0.56515	0.45703	0.28012	0.02949	-0.19166	-0.33417	-0.47669	-0.50618
55	0.59955	0.54549	0.43738	0.28012	0.04914	-0.17692	-0.31943	-0.47178	-0.49143
60	0.58972	0.55040	0.39315	0.28995	0.11303	-0.14743	-0.30469	-0.46195	-0.50126
65	0.58972	0.57006	0.38823	0.29486	0.08846	-0.13269	-0.32435	-0.47178	-0.50618
70	0.59955	0.55532	0.50126	0.22606	0.02457	-0.17692	-0.41280	-0.47669	-0.50618
75	0.59218	0.54549	0.48406	0.25555	0.00983	-0.19903	-0.41772	-0.47424	-0.50372
80	0.58431	0.53566	0.46686	0.28503	-0.00491	-0.22114	-0.42263	-0.47178	-0.50126
85	0.57252	0.52583	0.44475	0.27520	0.00246	-0.22606	-0.41035	-0.47178	-0.49389
90	0.56023	0.51600	0.42263	0.26537	0.00983	-0.23097	-0.39806	-0.47178	-0.48652

CONFIGURATION A

C_x

COEFFICIENT 5

BETA (DEGREES) ALPHA	-40	-30	-20	-10	0	+10	+20	+30	+40
0	-0.05475	-0.05475	-0.05475	-0.05475	-0.05475	-0.05475	-0.05475	-0.05475	-0.05475
5	-0.03939	-0.03939	-0.03939	-0.03939	-0.03939	-0.03939	-0.03939	-0.03939	-0.03939
10	-0.02404	-0.02404	-0.02404	-0.02404	-0.02404	-0.02404	-0.02404	-0.02404	-0.02404
15	-0.02604	-0.02604	-0.02604	-0.02604	-0.02604	-0.02604	-0.02604	-0.02604	-0.02604
20	-0.02804	-0.02804	-0.02804	-0.02804	-0.02804	-0.02804	-0.02804	-0.02804	-0.02804
25	-0.02069	-0.02670	-0.03005	-0.03205	-0.02404	-0.02670	-0.03538	-0.02988	-0.02203
30	-0.01335	-0.02537	-0.03205	-0.03605	-0.02003	-0.02537	-0.04273	-0.03472	-0.01602
35	0.00801	-0.01736	-0.02137	-0.02404	-0.01736	-0.02404	-0.03205	-0.02003	-0.00401
40	0.02604	0.00267	-0.01335	-0.01469	-0.01335	-0.02404	-0.02270	-0.00801	0.00534
45	0.02270	0.02270	-0.00134	-0.01202	-0.01870	-0.02003	0.0	-0.00134	0.00534
50	0.02671	0.03338	0.00801	-0.02003	-0.02003	-0.01335	0.01469	0.01469	0.02537
55	0.03739	0.02804	0.01068	-0.01068	-0.01202	0.0	0.02804	0.02671	0.02804
60	0.02804	0.02938	0.03205	0.01202	0.02003	0.02003	0.04273	0.03873	0.03739
65	0.03873	0.04407	0.04941	0.03338	0.03873	0.04407	0.04140	0.04273	0.04674
70	0.04407	0.04140	0.04674	0.03605	0.03472	0.03873	0.03739	0.04941	0.04807
75	0.04941	0.04374	0.04607	0.03605	0.03806	0.03806	0.04206	0.06459	0.05742
80	0.05475	0.05609	0.04540	0.03605	0.04140	0.03739	0.04674	0.07478	0.06677
85	0.05742	0.05676	0.05408	0.04503	0.04748	0.04874	0.06143	0.07211	0.07145
90	0.06009	0.05742	0.06276	0.05341	0.05475	0.06009	0.07612	0.06944	0.07612

CONFIGURATION A

C_z

COEFFICIENT 6

BETA (DEGREES) ALPHA	-40	-30	-20	-10	0	+10	+20	+30	+40
0	-0.05799	-0.05799	-0.05799	-0.05799	-0.05799	-0.05799	-0.05799	-0.05799	-0.05799
5	-0.43490	-0.43490	-0.43490	-0.43490	-0.43490	-0.43490	-0.43490	-0.43490	-0.43490
10	-0.81182	-0.81182	-0.81182	-0.81182	-0.81182	-0.81182	-0.81182	-0.81182	-0.81182
15	-1.22356	-1.22356	-1.22356	-1.22356	-1.22356	-1.22356	-1.22356	-1.22356	-1.22356
20	-1.63530	-1.63530	-1.63530	-1.63530	-1.63530	-1.63530	-1.63530	-1.63530	-1.63530
25	-1.48450	-1.64110	-1.73380	-1.87300	-2.08180	-1.95420	-1.75120	-1.67010	-1.53670
30	-1.33370	-1.64680	-1.83240	-2.11070	-2.52830	-2.27310	-1.86720	-1.71640	-1.43810
35	-1.51930	-1.75120	-1.98320	-2.29630	-2.84140	-2.42390	-1.97160	-1.71640	-1.46130
40	-1.69320	-1.85560	-2.12230	-2.48190	-2.92260	-2.57460	-2.06440	-1.78600	-1.48450
45	-1.69320	-1.98320	-2.23830	-2.53990	-2.91100	-2.58620	-2.22670	-1.79760	-1.55410
50	-1.71640	-2.07600	-2.23830	-2.44710	-2.76020	-2.45870	-2.23830	-1.82080	-1.57730
55	-1.80920	-2.02960	-2.16870	-2.38910	-2.62100	-2.37750	-2.09920	-1.90200	-1.65840
60	-1.80920	-2.12230	-2.31950	-2.41230	-2.56310	-2.41230	-2.23830	-1.97160	-1.71640
65	-1.91360	-2.19190	-2.35430	-2.43550	-2.59780	-2.45870	-2.23830	-1.97160	-1.78600
70	-2.04120	-2.21510	-2.30790	-2.50510	-2.59780	-2.40070	-2.19190	-2.07600	-1.86720
75	-2.08170	-2.25570	-2.37170	-2.49350	-2.57460	-2.45290	-2.30210	-2.15130	-1.91940
80	-2.12230	-2.29630	-2.43550	-2.48190	-2.55150	-2.50510	-2.41230	-2.22670	-1.97160
85	-2.12230	-2.26730	-2.39490	-2.48770	-2.57460	-2.48770	-2.40650	-2.22670	-1.98900
90	-2.12230	-2.23830	-2.35430	-2.49350	-2.59780	-2.47030	-2.40070	-2.22670	-2.00640

CONFIGURATION A

$C_{Y_{\delta r}}$

COEFFICIENT 7

BETA (DEGREES) ALPHA	-40	-30	-20	-10	0	+10	+20	+30	+40
0	0.00484	0.00484	0.00484	0.00484	0.00484	0.00484	0.00484	0.00484	0.00484
5	0.00464	0.00467	0.00467	0.00467	0.00467	0.00467	0.00467	0.00467	0.00467
10	0.00450	0.00450	0.00450	0.00450	0.00450	0.00450	0.00450	0.00450	0.00450
15	0.00449	0.00449	0.00449	0.00449	0.00449	0.00449	0.00449	0.00449	0.00449
20	0.00448	0.00448	0.00448	0.00448	0.00448	0.00448	0.00448	0.00448	0.00448
25	0.00299	0.00278	0.00299	0.00362	0.00481	0.00267	0.00262	0.00225	0.00253
30	0.00150	0.00109	0.00150	0.00277	0.00514	0.00085	0.00076	0.00002	0.00059
35	0.00148	0.00058	0.00098	0.00215	0.00431	0.00137	0.00062	-0.00105	0.00102
40	0.00146	0.00006	0.00045	0.00152	0.00347	0.00189	0.00048	-0.00211	0.00145
45	0.00080	0.00052	0.00042	0.00059	0.00123	0.00076	0.00043	-0.00099	0.00105
50	0.00014	0.00098	0.00038	-0.00034	-0.00102	-0.00038	0.00038	0.00013	0.00065
55	0.00063	0.00066	0.00056	-0.00051	-0.00070	-0.00122	-0.00030	0.00063	0.00047
60	-0.00020	0.00066	-0.00125	-0.00068	0.00143	-0.00058	0.00019	-0.00004	0.00081
65	0.00030	0.00148	-0.00041	0.00165	0.00261	0.00075	0.00154	-0.00021	0.00032
70	0.00146	0.00132	0.00202	0.00186	0.00132	0.00161	-0.00008	0.00046	0.00048
75	0.00155	0.00158	0.00220	0.00235	0.00125	0.00138	0.00026	0.00079	0.00073
80	0.00164	0.00183	0.00238	0.00283	0.00117	0.00114	0.00060	0.00113	0.00098
85	0.00123	0.00151	0.00173	0.00216	0.00142	0.00039	0.00034	0.00038	-0.00003
90	0.00082	0.00118	0.00107	0.00150	0.00166	-0.00036	0.00008	-0.00037	-0.00103

CONFIGURATION A

$C_{\ell_{\delta r}}$

COEFFICIENT 8

BETA (DEGREES) ALPHA	-40	-30	-20	-10	0	+10	+20	+30	+40
0	0.00017	0.00017	0.00017	0.00017	0.00017	0.00017	0.00017	0.00017	0.00017
5	0.00014	0.00014	0.00014	0.00014	0.00014	0.00014	0.00014	0.00014	0.00014
10	0.00011	0.00011	0.00011	0.00011	0.00011	0.00011	0.00011	0.00011	0.00011
15	0.00011	0.00011	0.00011	0.00011	0.00011	0.00011	0.00011	0.00011	0.00011
20	0.00010	0.00010	0.00010	0.00010	0.00010	0.00010	0.00010	0.00010	0.00010
25	0.00007	0.00004	0.00014	0.00014	0.00017	-0.00030	0.00004	-0.00010	0.00011
30	0.00004	-0.00003	0.00018	0.00017	0.00023	-0.00070	-0.00003	-0.00030	0.00012
35	-0.00013	0.0	0.00013	0.00023	-0.00006	-0.00028	0.00006	-0.00011	0.00011
40	-0.00029	0.00003	0.00007	0.00029	-0.00035	0.00014	0.00015	0.00008	0.00010
45	-0.00012	-0.00021	0.00006	0.00011	-0.00031	0.00008	0.00008	0.00008	-0.00001
50	0.00006	-0.00044	0.00005	-0.00007	-0.00026	0.00002	0.0	0.00001	-0.00012
55	0.00003	0.00002	0.00011	-0.00001	-0.00002	0.00007	0.00001	0.00009	0.00002
60	-0.00008	-0.00011	0.00007	0.00001	0.00004	0.00007	0.00001	0.00006	0.00007
65	-0.00010	0.00007	0.00010	0.00008	0.00009	-0.00005	-0.00005	0.00003	0.00004
70	0.00004	0.00005	0.00006	0.00008	0.00002	-0.00003	0.00005	0.00003	0.00005
75	0.00004	0.00002	0.00006	0.00013	-0.00004	0.00002	0.00005	0.00002	0.00005
80	0.00004	-0.00001	0.00005	0.00017	-0.00009	0.00006	0.00005	0.00001	0.00004
85	0.00001	0.00001	0.00006	0.00014	0.00003	0.00003	0.00001	0.00001	0.00004
90	-0.00002	0.00002	0.00008	0.00011	0.00015	-0.00001	-0.00003	0.0	0.00003

CONFIGURATION A
$C_{n_{\delta r}}$
COEFFICIENTS

BETA (DEGREES) ALPHA	-40	-30	-20	-10	0	+10	+20	+30	+40
0	-0.00135	-0.00135	-0.00135	-0.00135	-0.00135	-0.00135	-0.00135	-0.00135	-0.00135
5	-0.00132	-0.00132	-0.00132	-0.00132	-0.00132	-0.00132	-0.00132	-0.00132	-0.00132
10	-0.00128	-0.00128	-0.00128	-0.00128	-0.00128	-0.00128	-0.00128	-0.00128	-0.00128
15	-0.00127	-0.00127	-0.00127	-0.00127	-0.00127	-0.00127	-0.00127	-0.00127	-0.00127
20	-0.00126	-0.00126	-0.00126	-0.00126	-0.00126	-0.00126	-0.00126	-0.00126	-0.00126
25	-0.00080	-0.00084	-0.00062	-0.00112	-0.00130	-0.00115	-0.00079	-0.00068	-0.00036
30	-0.00035	-0.00043	0.00002	-0.00098	-0.00133	-0.00104	-0.00033	-0.00010	0.00054
35	-0.00038	-0.00012	-0.00017	-0.00009	-0.00136	-0.00097	-0.00050	0.00020	0.00040
40	-0.00040	0.00020	-0.00035	0.00081	-0.00138	-0.00089	-0.00067	0.00050	0.00026
45	-0.00036	0.00012	-0.00013	-0.00054	-0.00169	-0.00084	-0.00007	0.00044	0.00030
50	-0.00031	0.00003	0.00009	-0.00188	-0.00199	-0.00079	0.00053	0.00038	0.00033
55	-0.00022	-0.00074	-0.00122	-0.00240	-0.00122	-0.00037	0.00008	0.00015	0.00041
60	0.00053	-0.00157	-0.00271	-0.00152	0.00148	-0.00024	0.00058	0.00055	0.00005
65	0.00065	-0.00113	-0.00257	-0.00155	0.00244	0.00192	0.00180	0.0	-0.00020
70	-0.00070	0.00005	0.00176	0.00002	0.00003	0.00208	0.00104	-0.00039	-0.00043
75	-0.00058	0.00023	0.00073	-0.00007	0.00013	0.00099	0.00048	0.00051	-0.00022
80	-0.00047	-0.00050	-0.00029	-0.00017	0.00022	-0.00010	-0.00009	0.00141	0.0
85	-0.00036	-0.00047	-0.00039	-0.00013	-0.00005	-0.00001	0.00024	0.00079	0.00003
90	-0.00025	-0.00044	-0.00049	-0.00009	-0.00032	0.00009	0.00057	0.00017	0.00006

CONFIGURATION A

$C_{y_{\delta a}}$

COEFFICIENT 10

BETA (DEGREES) / ALPHA	-40	-30	-20	-10	0	+10	+20	+30	+40
0	0.00150	0.00150	0.00150	0.00150	0.00150	0.00150	0.00150	0.00150	0.00150
5	0.00151	0.00151	0.00151	0.00151	0.00151	0.00151	0.00151	0.00151	0.00151
10	0.00152	0.00152	0.00152	0.00152	0.00152	0.00152	0.00152	0.00152	0.00152
15	0.00104	0.00104	0.00104	0.00104	0.00104	0.00104	0.00104	0.00104	0.00104
20	0.00055	0.00055	0.00055	0.00055	0.00055	0.00055	0.00055	0.00055	0.00055
25	-0.00207	-0.00392	-0.00351	-0.00189	0.00031	0.00075	-0.00183	-0.00045	-0.00104
30	-0.00468	-0.00839	-0.00758	-0.00432	0.00008	0.00096	-0.00421	-0.00145	-0.00262
35	-0.00513	-0.00661	-0.00952	-0.00698	-0.00067	-0.00034	-0.00553	-0.00474	-0.00315
40	-0.00558	-0.00482	-0.01145	-0.00963	-0.00142	-0.00163	-0.00685	-0.00803	-0.00367
45	-0.00610	-0.00597	-0.00962	-0.00635	-0.00294	-0.00246	-0.00767	-0.00687	-0.00222
50	-0.00661	-0.00712	-0.00779	-0.00307	-0.00445	-0.00328	-0.00849	-0.00571	-0.00077
55	-0.00957	-0.00765	-0.00981	-0.00606	-0.00841	-0.01025	-0.01396	-0.00471	-0.00374
60	-0.00958	-0.01014	-0.00689	-0.01403	-0.01579	-0.00771	-0.01044	-0.00419	-0.00226
65	-0.00858	-0.01360	-0.01039	-0.01751	-0.01134	-0.00569	-0.01446	-0.00720	-0.00276
70	-0.00807	-0.00814	-0.01071	-0.01263	-0.00495	0.00023	-0.01061	-0.00472	-0.00177
75	-0.00708	-0.00840	-0.00949	-0.01284	-0.00522	-0.00254	-0.00712	-0.00346	-0.00201
80	-0.00610	-0.00867	-0.00827	-0.01304	-0.00550	-0.00532	-0.00364	-0.00221	-0.00226
85	-0.00562	-0.00793	-0.00705	-0.01081	-0.00648	-0.00383	-0.00262	-0.00221	-0.00274
90	-0.00514	-0.00720	-0.00584	-0.00858	-0.00747	-0.00235	-0.00160	-0.00221	-0.00323

CONFIGURATION A

$C_{\ell_{\delta a}}$

COEFFICIENT 11

BETA (DEGREES) ALPHA	-40	-30	-20	-10	0	+10	+20	+30	+40
0	-0.00160	-0.00160	-0.00160	-0.00160	-0.00160	-0.00160	-0.00160	-0.00160	-0.00160
5	-0.00160	-0.00160	-0.00160	-0.00160	-0.00160	-0.00160	-0.00160	-0.00160	-0.00160
10	-0.00159	-0.00159	-0.00159	-0.00159	-0.00159	-0.00159	-0.00159	-0.00159	-0.00159
15	-0.00175	-0.00175	-0.00175	-0.00175	-0.00175	-0.00175	-0.00175	-0.00175	-0.00175
20	-0.00191	-0.00191	-0.00191	-0.00191	-0.00191	-0.00191	-0.00191	-0.00191	-0.00191
25	-0.00095	-0.00054	-0.00081	-0.00121	-0.00169	-0.00095	-0.00167	-0.00108	-0.00111
30	0.00001	0.00082	0.00028	-0.00050	-0.00148	0.00001	-0.00142	-0.00025	-0.00031
35	0.00015	0.00038	0.00008	-0.00056	0.00036	0.00048	-0.00110	-0.00046	-0.00032
40	0.00029	-0.00006	-0.00012	-0.00063	0.00220	0.00094	-0.00077	-0.00067	-0.00033
45	0.00002	0.00017	-0.00021	-0.00027	0.00113	0.00022	-0.00064	-0.00038	-0.00081
50	-0.00025	0.00039	-0.00029	0.00010	0.00006	-0.00051	-0.00050	-0.00008	-0.00129
55	-0.00038	-0.00074	-0.00151	-0.00063	-0.00068	0.00005	0.00038	-0.00076	-0.00063
60	-0.00018	-0.00042	-0.00086	-0.00114	-0.00101	-0.00054	-0.00006	-0.00031	-0.00054
65	-0.00006	-0.00032	-0.00081	-0.00095	-0.00052	-0.00029	0.00027	-0.00012	-0.00065
70	-0.00032	-0.00039	-0.00029	-0.00125	-0.00086	-0.00051	0.00013	-0.00046	-0.00069
75	-0.00034	-0.00024	-0.00018	-0.00080	-0.00048	-0.00043	-0.00008	-0.00032	-0.00058
80	-0.00036	-0.00009	-0.00007	-0.00036	-0.00010	-0.00036	-0.00030	-0.00018	-0.00048
85	-0.00018	-0.00003	-0.00012	-0.00029	-0.00044	-0.00043	-0.00019	-0.00022	-0.00041
90	0.00001	0.00003	-0.00016	-0.00021	-0.00078	-0.00050	-0.00007	-0.00026	-0.00033

CONFIGURATION A

$C_{n_{\delta a}}$

COEFFICIENT 12

BETA (DEGREES) ALPHA	-40	-30	-20	-10	0	+10	+20	+30	+40
0	-0.00045	-0.00045	-0.00045	-0.00045	-0.00045	-0.00045	-0.00045	-0.00045	-0.00045
5	-0.00032	-0.00032	-0.00032	-0.00032	-0.00032	-0.00032	-0.00032	-0.00032	-0.00032
10	-0.00019	-0.00019	-0.00019	-0.00019	-0.00019	-0.00019	-0.00019	-0.00019	-0.00019
15	-0.00007	-0.00007	-0.00007	-0.00007	-0.00007	-0.00007	-0.00007	-0.00007	-0.00007
20	0.00004	0.00004	0.00004	0.00004	0.00004	0.00004	0.00004	0.00004	0.00004
25	0.00037	0.00090	0.00095	0.00062	0.00132	0.00070	-0.00044	0.00027	0.00041
30	0.00070	0.00177	0.00187	0.00121	0.00128	0.00137	-0.00092	0.00051	0.00078
35	0.00097	0.00150	0.00188	0.00081	0.00232	0.00304	0.00052	0.00036	0.00083
40	0.00123	0.00122	0.00188	0.00041	0.00336	0.00471	0.00195	0.00020	0.00087
45	0.00142	0.00267	0.00309	0.00474	0.00402	0.00319	-0.00046	0.00002	0.00175
50	0.00161	0.00412	0.00430	0.00907	0.00468	0.00167	-0.00286	-0.00016	0.00263
55	0.00317	0.00434	0.00690	0.00860	0.00221	-0.00042	-0.00392	0.00133	0.00186
60	0.00250	0.00740	0.00738	0.00066	-0.00436	-0.00132	-0.00431	0.00014	0.00205
65	-0.00004	0.00373	0.00508	-0.00013	-0.00275	-0.00111	0.00417	0.00205	0.00363
70	0.00212	-0.00002	0.00465	0.00187	0.00935	0.00870	0.00145	0.00110	0.00234
75	0.00214	0.00154	0.00359	0.00190	0.00523	0.00525	0.00099	-0.00129	0.00145
80	0.00216	0.00310	0.00253	0.00192	0.00111	0.00181	0.00054	-0.00368	0.00055
85	0.00190	0.00224	0.00265	0.00164	0.00214	0.00133	-0.00084	-0.00171	0.00072
90	0.00163	0.00137	0.00278	0.00136	0.00317	0.00085	-0.00223	0.00026	0.00089

CONFIGURATION A

$C_{z_{\delta e}}$

COEFFICIENT 13

BETA (DEGREES) ALPHA	-40	-30	-20	-10	0	+10	+20	+30	+40
0	-0.01943	-0.01943	-0.01943	-0.01943	-0.01943	-0.01943	-0.01943	-0.01943	-0.01943
5	-0.01997	-0.01997	-0.01997	-0.01997	-0.01997	-0.01997	-0.01997	-0.01997	-0.01997
10	-0.02052	-0.02052	-0.02052	-0.02052	-0.02052	-0.02052	-0.02052	-0.02052	-0.02052
15	-0.02044	-0.02044	-0.02044	-0.02044	-0.02044	-0.02044	-0.02044	-0.02044	-0.02044
20	-0.02036	-0.02036	-0.02036	-0.02036	-0.02036	-0.02036	-0.02036	-0.02036	-0.02036
25	-0.01282	-0.01652	-0.01673	-0.02014	-0.02675	-0.02391	-0.01582	-0.01594	-0.01868
30	-0.00528	-0.01267	-0.01310	-0.01993	-0.03313	-0.02746	-0.01129	-0.01151	-0.01700
35	-0.00424	-0.00668	-0.01149	-0.01799	-0.03000	-0.02378	-0.00973	-0.01026	-0.01137
40	-0.00319	-0.00068	-0.00988	-0.01605	-0.02687	-0.02010	-0.00816	-0.00900	-0.00574
45	-0.00229	-0.00353	-0.00717	-0.01193	-0.02193	-0.01425	-0.00867	-0.00604	-0.00422
50	-0.00139	-0.00638	-0.00446	-0.00781	-0.01698	-0.00839	-0.00918	-0.00307	-0.00270
55	-0.00047	-0.00695	-0.00767	-0.00813	-0.01559	-0.00871	-0.00830	-0.00537	-0.00352
60	0.00046	-0.00752	-0.01088	-0.00844	-0.01420	-0.00903	-0.00741	-0.00766	-0.00434
65	-0.00268	-0.00600	-0.00617	-0.00722	-0.01271	-0.00697	-0.00684	-0.00496	-0.00398
70	-0.00582	-0.00448	-0.00145	-0.00599	-0.01122	-0.00490	-0.00627	-0.00225	-0.00362
75	-0.00637	-0.00533	-0.00316	-0.00393	-0.00947	-0.00456	-0.00646	-0.00454	-0.00534
80	-0.00693	-0.00618	-0.00487	-0.00188	-0.00772	-0.00422	-0.00666	-0.00683	-0.00707
85	-0.00693	-0.00618	-0.00487	-0.00188	-0.00772	-0.00422	-0.00666	-0.00685	-0.00707
90	-0.00693	-0.00618	-0.00487	-0.00188	-0.00772	-0.00422	-0.00666	-0.00688	-0.00707

CONFIGURATION A

$C_{m_{\delta e}}$

COEFFICIENT 14

BETA (DEGREES) ALPHA	-40	-30	-20	-10	0	+10	+20	+30	+40
0	-0.03499	-0.03499	-0.03499	-0.03499	-0.03499	-0.03499	-0.03499	-0.03499	-0.03499
5	-0.03505	-0.03505	-0.03505	-0.03505	-0.03505	-0.03505	-0.03505	-0.03505	-0.03505
10	-0.03511	-0.03511	-0.03511	-0.03511	-0.03511	-0.03511	-0.03511	-0.03511	-0.03511
15	-0.03637	-0.03637	-0.03637	-0.03637	-0.03637	-0.03637	-0.03637	-0.03637	-0.03637
20	-0.03763	-0.03763	-0.03763	-0.03763	-0.03763	-0.03763	-0.03763	-0.03763	-0.03763
25	-0.02562	-0.02346	-0.02724	-0.03193	-0.03912	-0.03513	-0.02825	-0.02917	-0.01601
30	-0.01361	-0.00929	-0.01685	-0.02623	-0.04062	-0.03264	-0.01888	-0.02072	0.00562
35	-0.00856	-0.01511	-0.01301	-0.02081	-0.03550	-0.02654	-0.01624	-0.01021	0.00822
40	-0.00350	-0.02093	-0.00916	-0.01539	-0.03037	-0.02044	-0.01359	0.00030	0.01081
45	-0.00126	-0.00706	-0.00900	-0.01121	-0.02267	-0.01503	-0.00976	-0.00086	0.00835
50	0.00099	0.00682	-0.00883	-0.00702	-0.01497	-0.00962	-0.00592	-0.00201	0.00589
55	-0.00887	0.00911	0.00595	-0.00478	-0.00782	-0.00369	-0.00319	-0.00275	-0.00147
60	-0.01873	0.01140	0.02073	-0.00254	-0.00066	0.00224	-0.00045	-0.00349	-0.00882
65	-0.01389	-0.00308	0.00471	0.00230	0.00188	0.00694	0.00001	-0.00708	-0.01004
70	-0.00905	-0.01755	-0.01131	0.00713	0.00442	0.01163	0.00047	-0.01067	-0.01125
75	-0.00884	-0.01182	-0.01143	-0.00056	-0.00114	0.00123	-0.00460	-0.00843	-0.01213
80	-0.00864	-0.00608	-0.01155	-0.00825	-0.00670	-0.00917	-0.00968	-0.00619	-0.01301
85	-0.00864	-0.00608	-0.01155	-0.00825	-0.00670	-0.00917	-0.00968	-0.00619	-0.01301
90	-0.00864	-0.00608	-0.01155	-0.00825	-0.00670	-0.00917	-0.00968	-0.00619	-0.01301

CONFIGURATION A

$C_{x_{\delta e}}$

COEFFICIENT 15

BETA (DEGREES) ALPHA	-40	-30	-20	-10	0	+10	+20	+30	+40
0	0.00392	0.00392	0.00392	0.00392	0.00392	0.00392	0.00392	0.00392	0.00392
5	0.00295	0.00295	0.00295	0.00295	0.00295	0.00295	0.00295	0.00295	0.00295
10	0.00199	0.00199	0.00199	0.00199	0.00199	0.00199	0.00199	0.00199	0.00199
15	0.00131	0.00131	0.00131	0.00131	0.00131	0.00131	0.00131	0.00131	0.00131
20	0.00064	0.00064	0.00064	0.00064	0.00064	0.00064	0.00064	0.00064	0.00064
25	-0.00012	-0.00035	-0.00028	-0.00027	-0.00056	0.00030	-0.00034	-0.00055	-0.00029
30	-0.00087	-0.00134	-0.00119	-0.00126	-0.00175	-0.00004	-0.00132	-0.00174	-0.00121
35	-0.00147	-0.00193	-0.00215	-0.00226	-0.00267	-0.00181	-0.00232	-0.00196	-0.00153
40	-0.00206	-0.00252	-0.00311	-0.00325	-0.00359	-0.00358	-0.00331	-0.00217	-0.00184
45	-0.00257	-0.00274	-0.00353	-0.00410	-0.00420	-0.00410	-0.00340	-0.00266	-0.00206
50	-0.00308	-0.00295	-0.00395	-0.00494	-0.00481	-0.00461	-0.00348	-0.00314	-0.00227
55	-0.00342	-0.00353	-0.00356	-0.00459	-0.00439	-0.00480	-0.00339	-0.00298	-0.00265
60	-0.00376	-0.00411	-0.00316	-0.00423	-0.00396	-0.00498	-0.00330	-0.00282	-0.00303
65	-0.00374	-0.00435	-0.00381	-0.00441	-0.00445	-0.00465	-0.00361	-0.00358	-0.00341
70	-0.00371	-0.00459	-0.00446	-0.00459	-0.00493	-0.00432	-0.00391	-0.00433	-0.00378
75	-0.00378	-0.00429	-0.00466	-0.00506	-0.00527	-0.00486	-0.00452	-0.00390	-0.00372
80	-0.00386	-0.00399	-0.00487	-0.00554	-0.00561	-0.00540	-0.00514	-0.00347	-0.00366
85	-0.00386	-0.00399	-0.00487	-0.00554	-0.00561	-0.00540	-0.00514	-0.00347	-0.00366
90	-0.00386	-0.00399	-0.00487	-0.00554	-0.00561	-0.00540	-0.00514	-0.00347	-0.00366

CONFIGURATION A

ALPHA (DEGREES)	C_{y_p} COEF 16	C_{ℓ_p} COEF 17	C_{n_p} COEF 18	C_{y_r} COEF 19	C_{ℓ_r} COEF 20	C_{n_r} COEF 21	C_{z_q} COEF 22	C_{m_q} COEF 23	C_{x_q} COEF 24
0	-0.09003	-0.14991	-0.00938	0.67455	0.04427	-0.15449	-9.55210	-23.81200	0.07752
5	-0.08485	-0.15525	-0.00562	0.66426	0.08336	-0.16390	-7.34680	-23.53850	0.28324
10	-0.07966	-0.16058	-0.00187	0.65396	0.12244	-0.17330	-5.14150	-23.26500	0.48897
15	-0.14615	-0.18782	0.00215	0.78446	0.20907	-0.21182	-6.92840	-25.53000	0.85238
20	-0.21263	-0.21505	0.00617	0.91496	0.29570	-0.25034	-8.71730	-27.79500	1.21580
25	-0.18682	-0.28233	0.01500	0.85381	0.43143	-0.27332	-18.99710	-31.23900	2.10570
30	-0.16100	-0.34960	0.02382	0.79265	0.56725	-0.29630	-29.27700	-34.68300	3.09560
35	0.48037	-0.59684	0.01354	-0.05590	0.88361	-0.24858	-43.04300	-38.03200	3.08680
40	1.03980	-0.54280	0.00242	-1.49890	1.23420	-0.19695	-57.44300	-40.58000	3.38820
45	0.44946	-0.31589	-0.03497	-1.15690	0.80645	-0.04539	-66.26600	-39.73200	3.27050
50	0.02815	-0.15395	-0.05130	-0.96371	0.33907	0.21864	-67.85500	-29.96100	3.62100
55	-0.29888	-0.14661	-0.13921	0.80648	0.12032	0.86858	-56.70400	-25.28000	3.52100
60	-0.20904	-0.13798	-0.05000	0.07237	0.05703	0.30597	-49.20500	-21.55100	2.96470
65	-0.46779	-0.10665	-0.08551	0.49727	0.05928	0.40587	-39.26500	-16.13300	3.02950
70	-0.12746	-0.09241	-0.15002	-0.14060	-0.02969	0.22394	-29.30600	-13.84600	3.33160
75	0.17586	-0.11372	-0.20264	0.00522	-0.00206	0.09551	-19.86600	-7.35280	3.78090
80	0.47917	-0.14503	-0.25525	0.15104	0.02557	-0.03292	-10.42600	-0.85960	4.23020
85	0.38454	-0.14299	-0.25565	0.08058	0.00446	-0.10742	-5.82580	-8.37380	3.10200
90	0.28990	-0.14094	-0.25604	0.01011	-0.01665	-0.18192	-1.22560	-15.88800	1.97390

CONFIGURATION B

C_{ℓ}

COEFFICIENT 1

BETA (DEGREES) ALPHA	-40	-30	-20	-10	0	+10	+20	+30	+40
0	0.00480	0.00360	0.00240	0.00120	0.0	-0.00120	-0.00240	-0.00360	-0.00480
5	0.02400	0.01800	0.01200	0.00600	0.0	-0.00600	-0.01200	-0.01800	-0.02400
10	0.04600	0.03450	0.02300	0.01150	0.0	-0.01150	-0.02300	-0.03450	-0.04600
15	0.06000	0.04500	0.03000	0.01500	0.0	-0.01500	-0.03000	-0.04500	-0.06000
20	0.04320	0.03240	0.02160	0.01080	0.0	-0.01080	-0.02160	-0.03240	-0.04320
25	0.03680	0.02760	0.01840	0.00920	0.0	-0.00920	-0.01840	-0.02760	-0.03680
30	-0.03480	-0.02610	-0.01740	-0.00870	0.0	0.00870	0.01740	0.02610	0.03480
35	-0.00080	-0.00060	-0.00040	-0.00020	0.0	0.00020	0.00040	0.00060	0.00080
40	0.05000	0.03750	0.02500	0.01250	0.0	-0.01250	-0.02500	-0.03750	-0.05000
45	0.06400	0.04800	0.03200	0.01600	0.0	-0.01600	-0.03200	-0.04800	-0.06400
50	0.06800	0.05100	0.03400	0.01700	0.0	-0.01700	-0.03400	-0.05100	-0.06800
55	0.07200	0.05400	0.03600	0.01800	0.0	-0.01800	-0.03600	-0.05400	-0.07200
60	0.07200	0.05400	0.03600	0.01800	0.0	-0.01800	-0.03600	-0.05400	-0.07200
65	0.08120	0.06090	0.04060	0.02030	0.0	-0.02030	-0.04060	-0.06090	-0.08120
70	0.08600	0.06450	0.04300	0.02150	0.0	-0.02150	-0.04300	-0.06450	-0.08600
75	0.08680	0.06510	0.04340	0.02170	0.0	-0.02170	-0.04340	-0.06510	-0.08680
80	0.08400	0.06300	0.04200	0.02100	0.0	-0.02100	-0.04200	-0.06300	-0.08400
85	0.08160	0.06120	0.04080	0.02040	0.0	-0.02040	-0.04080	-0.06120	-0.08160
90	0.08000	0.06000	0.04000	0.02000	0.0	-0.02000	-0.04000	-0.06000	-0.08000

CONFIGURATION B

C_m

COEFFICIENT 2

BETA (DEGREES) ALPHA	-40	-30	-20	-10	0	+10	+20	+30	+40
0	-0.00350	-0.00350	-0.00350	-0.00350	-0.00350	-0.00350	-0.00350	-0.00350	-0.00350
5	-0.00490	-0.00490	-0.00490	-0.00490	-0.00490	-0.00490	-0.00490	-0.00490	-0.00490
10	-0.00900	-0.00900	-0.00900	-0.00900	-0.00900	-0.00900	-0.00900	-0.00900	-0.00900
15	-0.01480	-0.01480	-0.01480	-0.01480	-0.01480	-0.01480	-0.01480	-0.01480	-0.01480
20	-0.03500	-0.03500	-0.03500	-0.03500	-0.03500	-0.03500	-0.03500	-0.03500	-0.03500
25	-0.05800	-0.05800	-0.05800	-0.05800	-0.05800	-0.05800	-0.05800	-0.05800	-0.05800
30	-0.07900	-0.07900	-0.07900	-0.07900	-0.07900	-0.07900	-0.07900	-0.07900	-0.07900
35	-0.17150	-0.09160	-0.09160	-0.09160	-0.09160	-0.09160	-0.09160	-0.09160	-0.09160
40	-0.19000	-0.09550	-0.09550	-0.09550	-0.09550	-0.09550	-0.09550	-0.09550	-0.09550
45	-0.21130	-0.09030	-0.09030	-0.09030	-0.09030	-0.09030	-0.09030	-0.09030	-0.09030
50	-0.23100	-0.08730	-0.08730	-0.08730	-0.08730	-0.08730	-0.08730	-0.08730	-0.08730
55	0.0	-0.09950	-0.09950	-0.09950	-0.09950	-0.09950	-0.09950	-0.09950	-0.09950
60	0.0	-0.11830	-0.11830	-0.11830	-0.11830	-0.11830	-0.11830	-0.11830	-0.11830
65	0.0	-0.13080	-0.13080	-0.13080	-0.13080	-0.13080	-0.13080	-0.13080	-0.13080
70	0.0	-0.14700	-0.14700	-0.14700	-0.14700	-0.14700	-0.14700	-0.14700	-0.14700
75	-0.09160	-0.17150	-0.17150	-0.17150	-0.17150	-0.17150	-0.17150	-0.17150	-0.17150
80	-0.09550	-0.19000	-0.19000	-0.19000	-0.19000	-0.19000	-0.19000	-0.19000	-0.19000
85	-0.09030	-0.21130	-0.21130	-0.21130	-0.21130	-0.21130	-0.21130	-0.21130	-0.21130
90	-0.08730	-0.23100	-0.23100	-0.23100	-0.23100	-0.23100	-0.23100	-0.23100	-0.23100

CONFIGURATION B

C_n

COEFFICIENT 3

BETA (DEGREES) ALPHA	-40	-30	-20	-10	0	+10	+20	+30	+40
0	-0.04000	-0.03000	-0.02000	-0.01000	0.0	0.01000	0.02000	0.03000	0.04000
5	-0.04000	-0.03000	-0.02000	-0.01000	0.0	0.01000	0.02000	0.03000	0.04000
10	-0.03600	-0.02700	-0.01800	-0.00900	0.0	0.00900	0.01800	0.02700	0.03600
15	-0.02400	-0.01800	-0.01200	-0.00600	0.0	0.00600	0.01200	0.01800	0.02400
20	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
25	0.04000	0.03000	0.02000	0.01000	0.0	-0.01000	-0.02000	-0.03000	-0.04000
30	0.09200	0.06900	0.04600	0.02300	0.0	-0.02300	-0.04600	-0.06900	-0.09200
35	0.09200	0.06900	0.04600	0.02300	0.0	-0.02300	-0.04600	-0.06900	-0.09200
40	0.08800	0.06600	0.04400	0.02200	0.0	-0.02200	-0.04400	-0.06600	-0.08800
45	0.07200	0.05400	0.03600	0.01800	0.0	-0.01800	-0.03600	-0.05400	-0.07200
50	0.06000	0.04500	0.03000	0.01500	0.0	-0.01500	-0.03000	-0.04500	-0.06000
55	0.05200	0.03900	0.02600	0.01300	0.0	-0.01300	-0.02600	-0.03900	-0.05200
60	0.05200	0.03900	0.02600	0.01300	0.0	-0.01300	-0.02600	-0.03900	-0.05200
65	0.04800	0.03600	0.02400	0.01200	0.0	-0.01200	-0.02400	-0.03600	-0.04800
70	0.04400	0.03300	0.02200	0.01100	0.0	-0.01100	-0.02200	-0.03300	-0.04400
75	0.04400	0.03300	0.02200	0.01100	0.0	-0.01100	-0.02200	-0.03300	-0.04400
80	0.04800	0.03600	0.02400	0.01200	0.0	-0.01200	-0.02400	-0.03600	-0.04800
85	0.04800	0.03600	0.02400	0.01200	0.0	-0.01200	-0.02400	-0.03600	-0.04800
90	0.04400	0.03300	0.02200	0.01100	0.0	-0.01100	-0.02200	-0.03300	-0.04400

CONFIGURATION B

C_y

COEFFICIENT 4

BETA (DEGREES) ALPHA	-40	-30	-20	-10	0	+10	+20	+30	+40
0	0.28000	0.21000	0.14000	0.07000	0.0	-0.07000	-0.14000	-0.21000	-0.28000
5	0.32000	0.24000	0.16000	0.08000	0.0	-0.08000	-0.16000	-0.24000	-0.32000
10	0.34000	0.25500	0.17000	0.08500	0.0	-0.08500	-0.17000	-0.25500	-0.34000
15	0.34000	0.25500	0.17000	0.08500	0.0	-0.08500	-0.17000	-0.25500	-0.34000
20	0.32000	0.24000	0.16000	0.08000	0.0	-0.08000	-0.16000	-0.24000	-0.32000
25	0.28000	0.21000	0.14000	0.07000	0.0	-0.07000	-0.14000	-0.21000	-0.28000
30	0.22400	0.16800	0.11200	0.05600	0.0	-0.05600	-0.11200	-0.16800	-0.22400
35	0.13600	0.10200	0.06800	0.03400	0.0	-0.03400	-0.06800	-0.10200	-0.13600
40	0.04000	0.03000	0.02000	0.01000	0.0	-0.01000	-0.02000	-0.03000	-0.04000
45	0.07200	0.05400	0.03600	0.01800	0.0	-0.01800	-0.03600	-0.05400	-0.07200
50	0.10000	0.07500	0.05000	0.02500	0.0	-0.02500	-0.05000	-0.07500	-0.10000
55	0.08000	0.06000	0.04000	0.02000	0.0	-0.02000	-0.04000	-0.06000	-0.08000
60	0.06000	0.04500	0.03000	0.01500	0.0	-0.01500	-0.03000	-0.04500	-0.06000
65	0.04800	0.03600	0.02400	0.01200	0.0	-0.01200	-0.02400	-0.03600	-0.04800
70	0.04000	0.03000	0.02000	0.01000	0.0	-0.01000	-0.02000	-0.03000	-0.04000
75	0.06000	0.04500	0.03000	0.01500	0.0	-0.01500	-0.03000	-0.04500	-0.06000
80	0.07600	0.05700	0.03800	0.01900	0.0	-0.01900	-0.03800	-0.05700	-0.07600
85	0.08000	0.06000	0.04000	0.02000	0.0	-0.02000	-0.04000	-0.06000	-0.08000
90	0.08000	0.06000	0.04000	0.02000	0.0	-0.02000	-0.04000	-0.06000	-0.08000

CONFIGURATION B

C_x

COEFFICIENT 5

BETA (DEGREES) ALPHA	-40	-30	-20	-10	0	+10	+20	+30	+40
0	-0.03330	-0.03330	-0.03330	-0.03330	-0.03330	-0.03330	-0.03330	-0.03330	-0.03330
5	-0.01310	-0.01310	-0.01310	-0.01310	-0.01310	-0.01310	-0.01310	-0.01310	-0.01310
10	-0.01290	-0.01290	-0.01290	-0.01290	-0.01290	-0.01290	-0.01290	-0.01290	-0.01290
15	0.00670	0.00670	0.00670	0.00670	0.00670	0.00670	0.00670	0.00670	0.00670
20	0.00630	0.00630	0.00630	0.00630	0.00630	0.00630	0.00630	0.00630	0.00630
25	0.00500	0.00500	0.00500	0.00500	0.00500	0.00500	0.00500	0.00500	0.00500
30	0.01760	0.01760	0.01760	0.01760	0.01760	0.01760	0.01760	0.01760	0.01760
35	-0.00960	-0.00960	-0.00960	-0.00960	-0.00960	-0.00960	-0.00960	-0.00960	-0.00960
40	-0.01850	-0.01850	-0.01850	-0.01850	-0.01850	-0.01850	-0.01850	-0.01850	-0.01850
45	-0.01920	-0.01920	-0.01920	-0.01920	-0.01920	-0.01920	-0.01920	-0.01920	-0.01920
50	-0.01720	-0.01720	-0.01720	-0.01720	-0.01720	-0.01720	-0.01720	-0.01720	-0.01720
55	0.00140	0.00140	0.00140	0.00140	0.00140	0.00140	0.00140	0.00140	0.00140
60	0.01360	0.01860	0.01860	0.01860	0.01860	0.01860	0.01860	0.01860	0.01860
65	0.01810	0.01810	0.01810	0.01810	0.01810	0.01810	0.01810	0.01810	0.01810
70	0.01870	0.01870	0.01870	0.01870	0.01870	0.01870	0.01870	0.01870	0.01870
75	0.03380	0.03380	0.03380	0.03380	0.03380	0.03380	0.03380	0.03380	0.03380
80	0.03510	0.03510	0.03510	0.03510	0.03510	0.03510	0.03510	0.03510	0.03510
85	0.03680	0.03880	0.03880	0.03880	0.03880	0.03880	0.03880	0.03880	0.03880
90	0.03300	0.03300	0.03300	0.03300	0.03300	0.03300	0.03300	0.03300	0.03300

CONFIGURATION B

C_z

COEFFICIENT 6

BETA (DEGREES) ALPHA	-40	-30	-20	-10	0	+10	+20	+30	+40
0	0.02000	0.02000	0.02000	0.02000	0.02000	0.02000	0.02000	0.02000	0.02000
5	-0.18900	-0.18900	-0.18900	-0.18900	-0.18900	-0.18900	-0.18900	-0.18900	-0.18900
10	-0.43000	-0.43000	-0.43000	-0.43000	-0.43000	-0.43000	-0.43000	-0.43000	-0.43000
15	-0.69100	-0.69100	-0.69100	-0.69100	-0.69100	-0.69100	-0.69100	-0.69100	-0.69100
20	-0.94800	-0.94800	-0.94800	-0.94800	-0.94800	-0.94800	-0.94800	-0.94800	-0.94800
25	-1.14400	-1.14400	-1.14400	-1.14400	-1.14400	-1.14400	-1.14400	-1.14400	-1.14400
30	-1.26900	-1.26900	-1.26900	-1.26900	-1.26900	-1.26900	-1.26900	-1.26900	-1.26900
35	-1.32000	-1.32000	-1.32000	-1.32000	-1.32000	-1.32000	-1.32000	-1.32000	-1.32000
40	-1.26800	-1.26800	-1.26800	-1.26800	-1.26800	-1.26800	-1.26800	-1.26800	-1.26800
45	-1.20100	-1.20100	-1.20100	-1.20100	-1.20100	-1.20100	-1.20100	-1.20100	-1.20100
50	-1.17500	-1.17500	-1.17500	-1.17500	-1.17500	-1.17500	-1.17500	-1.17500	-1.17500
55	-1.20500	-1.20500	-1.20500	-1.20500	-1.20500	-1.20500	-1.20500	-1.20500	-1.20500
60	-1.25600	-1.25600	-1.25600	-1.25600	-1.25600	-1.25600	-1.25600	-1.25600	-1.25600
65	-1.29300	-1.29300	-1.29300	-1.29300	-1.29300	-1.29300	-1.29300	-1.29300	-1.29300
70	-1.34600	-1.34600	-1.34600	-1.34600	-1.34600	-1.34600	-1.34600	-1.34600	-1.34600
75	-1.38800	-1.38800	-1.38800	-1.38800	-1.38800	-1.38800	-1.38800	-1.38800	-1.38800
80	-1.41600	-1.41600	-1.41600	-1.41600	-1.41600	-1.41600	-1.41600	-1.41600	-1.41600
85	-1.42200	-1.42200	-1.42200	-1.42200	-1.42200	-1.42200	-1.42200	-1.42200	-1.42200
90	-1.41700	-1.41700	-1.41700	-1.41700	-1.41700	-1.41700	-1.41700	-1.41700	-1.41700

CONFIGURATION B

$C_{y\delta r}$

COEFFICIENT 7

BETA (DEGREES) ALPHA	-40	-30	-20	-10	0	+10	+20	+30	+40
0	0.00160	0.00160	0.00160	0.00160	0.00160	0.00160	0.00160	0.00160	0.00160
5	0.00160	0.00160	0.00160	0.00160	0.00160	0.00160	0.00160	0.00160	0.00160
10	0.00160	0.00160	0.00160	0.00160	0.00160	0.00160	0.00160	0.00160	0.00160
15	0.00148	0.00148	0.00148	0.00148	0.00148	0.00148	0.00148	0.00148	0.00148
20	0.00132	0.00132	0.00132	0.00132	0.00132	0.00132	0.00132	0.00132	0.00132
25	0.00120	0.00120	0.00120	0.00120	0.00120	0.00120	0.00120	0.00120	0.00120
30	0.00120	0.00120	0.00120	0.00120	0.00120	0.00120	0.00120	0.00120	0.00120
35	0.00112	0.00112	0.00112	0.00112	0.00112	0.00112	0.00112	0.00112	0.00112
40	0.00088	0.00088	0.00088	0.00088	0.00088	0.00088	0.00088	0.00088	0.00088
45	0.00040	0.00040	0.00040	0.00040	0.00040	0.00040	0.00040	0.00040	0.00040
50	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
55	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
60	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
65	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
70	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
75	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
80	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
85	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
90	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0

CONFIGURATION B

$C_{\ell_{\delta r}}$

COEFFICIENT 8

BETA (DEGREES) ALPHA	-40	-30	-20	-10	0	+10	+20	+30	+40
0	0.00008	0.00008	0.00008	0.00008	0.00008	0.00008	0.00008	0.00008	0.00008
5	0.00007	0.00007	0.00007	0.00007	0.00007	0.00007	0.00007	0.00007	0.00007
10	0.00008	0.00008	0.00008	0.00008	0.00008	0.00008	0.00008	0.00008	0.00008
15	0.00011	0.00011	0.00011	0.00011	0.00011	0.00011	0.00011	0.00011	0.00011
20	0.00018	0.00018	0.00018	0.00018	0.00018	0.00018	0.00018	0.00018	0.00018
25	0.00028	0.00028	0.00028	0.00028	0.00028	0.00028	0.00028	0.00028	0.00028
30	0.00038	0.00038	0.00038	0.00038	0.00038	0.00038	0.00038	0.00038	0.00038
35	0.00044	0.00044	0.00044	0.00044	0.00044	0.00044	0.00044	0.00044	0.00044
40	0.00039	0.00039	0.00039	0.00039	0.00039	0.00039	0.00039	0.00039	0.00039
45	0.00011	0.00011	0.00011	0.00011	0.00011	0.00011	0.00011	0.00011	0.00011
50	-0.00003	-0.00003	-0.00003	-0.00003	-0.00003	-0.00003	-0.00003	-0.00003	-0.00003
55	-0.00003	-0.00003	-0.00003	-0.00003	-0.00003	-0.00003	-0.00003	-0.00003	-0.00003
60	-0.00001	-0.00001	-0.00001	-0.00001	-0.00001	-0.00001	-0.00001	-0.00001	-0.00001
65	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
70	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
75	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
80	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
85	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
90	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0

CONFIGURATION B

$C_{n_{\delta r}}$

COEFFICIENT 9

BETA (DEGREES) ALPHA	-40	-30	-20	-10	0	+10	+20	+30	+40
0	-0.00052	-0.00052	-0.00052	-0.00052	-0.00052	-0.00052	-0.00052	-0.00052	-0.00052
5	-0.00053	-0.00053	-0.00053	-0.00053	-0.00053	-0.00053	-0.00053	-0.00053	-0.00053
10	-0.00056	-0.00056	-0.00056	-0.00056	-0.00056	-0.00056	-0.00056	-0.00056	-0.00056
15	-0.00059	-0.00059	-0.00059	-0.00059	-0.00059	-0.00059	-0.00059	-0.00059	-0.00059
20	-0.00064	-0.00064	-0.00064	-0.00064	-0.00064	-0.00064	-0.00064	-0.00064	-0.00064
25	-0.00070	-0.00070	-0.00070	-0.00070	-0.00070	-0.00070	-0.00070	-0.00070	-0.00070
30	-0.00074	-0.00074	-0.00074	-0.00074	-0.00074	-0.00074	-0.00074	-0.00074	-0.00074
35	-0.00040	-0.00040	-0.00040	-0.00040	-0.00040	-0.00040	-0.00040	-0.00040	-0.00040
40	-0.00013	-0.00013	-0.00013	-0.00013	-0.00013	-0.00013	-0.00013	-0.00013	-0.00013
45	-0.00004	-0.00004	-0.00004	-0.00004	-0.00004	-0.00004	-0.00004	-0.00004	-0.00004
50	-0.00002	-0.00002	-0.00002	-0.00002	-0.00002	-0.00002	-0.00002	-0.00002	-0.00002
55	-0.00003	-0.00003	-0.00003	-0.00003	-0.00003	-0.00003	-0.00003	-0.00003	-0.00003
60	-0.00004	-0.00004	-0.00004	-0.00004	-0.00004	-0.00004	-0.00004	-0.00004	-0.00004
65	-0.00004	-0.00004	-0.00004	-0.00004	-0.00004	-0.00004	-0.00004	-0.00004	-0.00004
70	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
75	0.00004	0.00004	0.00004	0.00004	0.00004	0.00004	0.00004	0.00004	0.00004
80	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
85	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
90	0.00007	0.00007	0.00007	0.00007	0.00007	0.00007	0.00007	0.00007	0.00007

CONFIGURATION B

$C_{y_{\delta a}}$

COEFFICIENT 10

BETA (DEGREES) ALPHA	-40	-30	-20	-10	0	+10	+20	+30	+40
0	0.00214	0.00214	0.00214	0.00214	0.00214	0.00214	0.00214	0.00214	0.00214
5	0.00214	0.00214	0.00214	0.00214	0.00214	0.00214	0.00214	0.00214	0.00214
10	0.00214	0.00214	0.00214	0.00214	0.00214	0.00214	0.00214	0.00214	0.00214
15	0.00243	0.00243	0.00243	0.00243	0.00243	0.00243	0.00243	0.00243	0.00243
20	0.00229	0.00229	0.00229	0.00229	0.00229	0.00229	0.00229	0.00229	0.00229
25	0.00143	0.00143	0.00143	0.00143	0.00143	0.00143	0.00143	0.00143	0.00143
30	0.00071	0.00071	0.00071	0.00071	0.00071	0.00071	0.00071	0.00071	0.00071
35	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
40	-0.00057	-0.00057	-0.00057	-0.00057	-0.00057	-0.00057	-0.00057	-0.00057	-0.00057
45	-0.00071	-0.00071	-0.00071	-0.00071	-0.00071	-0.00071	-0.00071	-0.00071	-0.00071
50	-0.00071	-0.00071	-0.00071	-0.00071	-0.00071	-0.00071	-0.00071	-0.00071	-0.00071
55	-0.00043	-0.00043	-0.00043	-0.00043	-0.00043	-0.00043	-0.00043	-0.00043	-0.00043
60	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
65	0.00029	0.00029	0.00029	0.00029	0.00029	0.00029	0.00029	0.00029	0.00029
70	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
75	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
80	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
85	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
90	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0

CONFIGURATION B

$C_{\ell_{\delta a}}$

COEFFICIENT 11

BETA (DEGREES) ALPHA	-40	-30	-20	-10	0	+10	+20	+30	+40
0	-0.00200	-0.00200	-0.00200	-0.00200	-0.00200	-0.00200	-0.00200	-0.00200	-0.00200
5	-0.00209	-0.00209	-0.00209	-0.00209	-0.00209	-0.00209	-0.00209	-0.00209	-0.00209
10	-0.00214	-0.00214	-0.00214	-0.00214	-0.00214	-0.00214	-0.00214	-0.00214	-0.00214
15	-0.00213	-0.00213	-0.00213	-0.00213	-0.00213	-0.00213	-0.00213	-0.00213	-0.00213
20	-0.00193	-0.00193	-0.00193	-0.00193	-0.00193	-0.00193	-0.00193	-0.00193	-0.00193
25	-0.00157	-0.00157	-0.00157	-0.00157	-0.00157	-0.00157	-0.00157	-0.00157	-0.00157
30	-0.00086	-0.00086	-0.00086	-0.00086	-0.00086	-0.00086	-0.00086	-0.00086	-0.00086
35	-0.00057	-0.00057	-0.00057	-0.00057	-0.00057	-0.00057	-0.00057	-0.00057	-0.00057
40	-0.00057	-0.00057	-0.00057	-0.00057	-0.00057	-0.00057	-0.00057	-0.00057	-0.00057
45	-0.00066	-0.00066	-0.00066	-0.00066	-0.00066	-0.00066	-0.00066	-0.00066	-0.00066
50	-0.00071	-0.00071	-0.00071	-0.00071	-0.00071	-0.00071	-0.00071	-0.00071	-0.00071
55	-0.00060	-0.00060	-0.00060	-0.00060	-0.00060	-0.00060	-0.00060	-0.00060	-0.00060
60	-0.00057	-0.00057	-0.00057	-0.00057	-0.00057	-0.00057	-0.00057	-0.00057	-0.00057
65	-0.00049	-0.00049	-0.00049	-0.00049	-0.00049	-0.00049	-0.00049	-0.00049	-0.00049
70	-0.00043	-0.00043	-0.00043	-0.00043	-0.00043	-0.00043	-0.00043	-0.00043	-0.00043
75	-0.00031	-0.00031	-0.00031	-0.00031	-0.00031	-0.00031	-0.00031	-0.00031	-0.00031
80	-0.00029	-0.00029	-0.00029	-0.00029	-0.00029	-0.00029	-0.00029	-0.00029	-0.00029
85	-0.00029	-0.00029	-0.00029	-0.00029	-0.00029	-0.00029	-0.00029	-0.00029	-0.00029
90	-0.00026	-0.00026	-0.00026	-0.00026	-0.00026	-0.00026	-0.00026	-0.00026	-0.00026

CONFIGURATION B

$C_{n_{\delta a}}$

COEFFICIENT 12

BETA (DEGREES) ALPHA	-40	-30	-20	-10	0	+10	+20	+30	+40
0	-0.00050	-0.00050	-0.00050	-0.00050	-0.00050	-0.00050	-0.00050	-0.00050	-0.00050
5	-0.00057	-0.00057	-0.00057	-0.00057	-0.00057	-0.00057	-0.00057	-0.00057	-0.00057
10	-0.00059	-0.00059	-0.00059	-0.00059	-0.00059	-0.00059	-0.00059	-0.00059	-0.00059
15	-0.00051	-0.00051	-0.00051	-0.00051	-0.00051	-0.00051	-0.00051	-0.00051	-0.00051
20	-0.00043	-0.00043	-0.00043	-0.00043	-0.00043	-0.00043	-0.00043	-0.00043	-0.00043
25	-0.00041	-0.00041	-0.00041	-0.00041	-0.00041	-0.00041	-0.00041	-0.00041	-0.00041
30	-0.00027	-0.00027	-0.00027	-0.00027	-0.00027	-0.00027	-0.00027	-0.00027	-0.00027
35	-0.00021	-0.00021	-0.00021	-0.00021	-0.00021	-0.00021	-0.00021	-0.00021	-0.00021
40	-0.00011	-0.00011	-0.00011	-0.00011	-0.00011	-0.00011	-0.00011	-0.00011	-0.00011
45	-0.00003	-0.00003	-0.00003	-0.00003	-0.00003	-0.00003	-0.00003	-0.00003	-0.00003
50	0.00004	0.00004	0.00004	0.00004	0.00004	0.00004	0.00004	0.00004	0.00004
55	0.00013	0.00013	0.00013	0.00013	0.00013	0.00013	0.00013	0.00013	0.00013
60	0.00019	0.00019	0.00019	0.00019	0.00019	0.00019	0.00019	0.00019	0.00019
65	0.00026	0.00026	0.00026	0.00026	0.00026	0.00026	0.00026	0.00026	0.00026
70	0.00031	0.00031	0.00031	0.00031	0.00031	0.00031	0.00031	0.00031	0.00031
75	0.00031	0.00031	0.00031	0.00031	0.00031	0.00031	0.00031	0.00031	0.00031
80	0.00023	0.00023	0.00023	0.00023	0.00023	0.00023	0.00023	0.00023	0.00023
85	0.00024	0.00024	0.00024	0.00024	0.00024	0.00024	0.00024	0.00024	0.00024
90	0.00029	0.00029	0.00029	0.00029	0.00029	0.00029	0.00029	0.00029	0.00029

CONFIGURATION B

$C_{z_{\delta e}}$

COEFFICIENT 13

BETA (DEGREES) ALPHA	-40	-30	-20	-10	0	+10	+20	+30	+40
0	-0.00924	-0.00924	-0.00924	-0.00924	-0.00924	-0.00924	-0.00924	-0.00924	-0.00924
5	-0.00957	-0.00957	-0.00957	-0.00957	-0.00957	-0.00957	-0.00957	-0.00957	-0.00957
10	-0.01005	-0.01005	-0.01005	-0.01005	-0.01005	-0.01005	-0.01005	-0.01005	-0.01005
15	-0.01092	-0.01092	-0.01092	-0.01092	-0.01092	-0.01092	-0.01092	-0.01092	-0.01092
20	-0.01200	-0.01200	-0.01200	-0.01200	-0.01200	-0.01200	-0.01200	-0.01200	-0.01200
25	-0.01135	-0.01135	-0.01135	-0.01135	-0.01135	-0.01135	-0.01135	-0.01135	-0.01135
30	-0.00811	-0.00811	-0.00811	-0.00811	-0.00811	-0.00811	-0.00811	-0.00811	-0.00811
35	-0.00762	-0.00762	-0.00762	-0.00762	-0.00762	-0.00762	-0.00762	-0.00762	-0.00762
40	-0.00735	-0.00735	-0.00735	-0.00735	-0.00735	-0.00735	-0.00735	-0.00735	-0.00735
45	-0.00622	-0.00622	-0.00622	-0.00622	-0.00622	-0.00622	-0.00622	-0.00622	-0.00622
50	-0.00508	-0.00508	-0.00508	-0.00508	-0.00508	-0.00508	-0.00508	-0.00508	-0.00508
55	-0.00541	-0.00541	-0.00541	-0.00541	-0.00541	-0.00541	-0.00541	-0.00541	-0.00541
60	-0.00551	-0.00551	-0.00551	-0.00551	-0.00551	-0.00551	-0.00551	-0.00551	-0.00551
65	-0.00476	-0.00476	-0.00476	-0.00476	-0.00476	-0.00476	-0.00476	-0.00476	-0.00476
70	-0.00405	-0.00405	-0.00405	-0.00405	-0.00405	-0.00405	-0.00405	-0.00405	-0.00405
75	-0.00400	-0.00400	-0.00400	-0.00400	-0.00400	-0.00400	-0.00400	-0.00400	-0.00400
80	-0.00400	-0.00400	-0.00400	-0.00400	-0.00400	-0.00400	-0.00400	-0.00400	-0.00400
85	-0.00335	-0.00335	-0.00335	-0.00335	-0.00335	-0.00335	-0.00335	-0.00335	-0.00335
90	-0.00265	-0.00265	-0.00265	-0.00265	-0.00265	-0.00265	-0.00265	-0.00265	-0.00265

CONFIGURATION B

$C_{m_{\delta e}}$

COEFFICIENT 14

BETA (DEGREES) ALPHA	-40	-30	-20	-10	0	+10	+20	+30	+40
0	-0.00362	-0.00362	-0.00362	-0.00362	-0.00362	-0.00362	-0.00362	-0.00362	-0.00362
5	-0.00382	-0.00382	-0.00382	-0.00382	-0.00382	-0.00382	-0.00382	-0.00382	-0.00382
10	-0.00400	-0.00400	-0.00400	-0.00400	-0.00400	-0.00400	-0.00400	-0.00400	-0.00400
15	-0.00411	-0.00411	-0.00411	-0.00411	-0.00411	-0.00411	-0.00411	-0.00411	-0.00411
20	-0.00416	-0.00416	-0.00416	-0.00416	-0.00416	-0.00416	-0.00416	-0.00416	-0.00416
25	-0.00416	-0.00416	-0.00416	-0.00416	-0.00416	-0.00416	-0.00416	-0.00416	-0.00416
30	-0.00368	-0.00368	-0.00368	-0.00368	-0.00368	-0.00368	-0.00368	-0.00368	-0.00368
35	-0.00348	-0.00348	-0.00348	-0.00348	-0.00348	-0.00348	-0.00348	-0.00348	-0.00348
40	-0.00332	-0.00332	-0.00332	-0.00332	-0.00332	-0.00332	-0.00332	-0.00332	-0.00332
45	-0.00297	-0.00297	-0.00297	-0.00297	-0.00297	-0.00297	-0.00297	-0.00297	-0.00297
50	-0.00265	-0.00265	-0.00265	-0.00265	-0.00265	-0.00265	-0.00265	-0.00265	-0.00265
55	-0.00254	-0.00254	-0.00254	-0.00254	-0.00254	-0.00254	-0.00254	-0.00254	-0.00254
60	-0.00249	-0.00249	-0.00249	-0.00249	-0.00249	-0.00249	-0.00249	-0.00249	-0.00249
65	-0.00243	-0.00243	-0.00243	-0.00243	-0.00243	-0.00243	-0.00243	-0.00243	-0.00243
70	-0.00232	-0.00232	-0.00232	-0.00232	-0.00232	-0.00232	-0.00232	-0.00232	-0.00232
75	-0.00208	-0.00208	-0.00208	-0.00208	-0.00208	-0.00208	-0.00208	-0.00208	-0.00208
80	-0.00184	-0.00184	-0.00184	-0.00184	-0.00184	-0.00184	-0.00184	-0.00184	-0.00184
85	-0.00173	-0.00173	-0.00173	-0.00173	-0.00173	-0.00173	-0.00173	-0.00173	-0.00173
90	-0.00168	-0.00168	-0.00168	-0.00168	-0.00168	-0.00168	-0.00168	-0.00168	-0.00168

CONFIGURATION 8

$C_{x_{\delta e}}$

COEFFICIENT 15

BETA (DEGREES) ALPHA	-40	-30	-20	-10	0	+10	+20	+30	+40
0	0.00102	0.00102	0.00102	0.00102	0.00102	0.00102	0.00102	0.00102	0.00102
5	0.00106	0.00106	0.00106	0.00106	0.00106	0.00106	0.00106	0.00106	0.00106
10	0.00108	0.00108	0.00108	0.00108	0.00108	0.00108	0.00108	0.00108	0.00108
15	0.00104	0.00104	0.00104	0.00104	0.00104	0.00104	0.00104	0.00104	0.00104
20	0.00095	0.00095	0.00095	0.00095	0.00095	0.00095	0.00095	0.00095	0.00095
25	0.00077	0.00077	0.00077	0.00077	0.00077	0.00077	0.00077	0.00077	0.00077
30	0.00039	0.00039	0.00039	0.00039	0.00039	0.00039	0.00039	0.00039	0.00039
35	0.00035	0.00035	0.00035	0.00035	0.00035	0.00035	0.00035	0.00035	0.00035
40	0.00039	0.00039	0.00039	0.00039	0.00039	0.00039	0.00039	0.00039	0.00039
45	0.00043	0.00043	0.00043	0.00043	0.00043	0.00043	0.00043	0.00043	0.00043
50	0.00044	0.00044	0.00044	0.00044	0.00044	0.00044	0.00044	0.00044	0.00044
55	0.00036	0.00036	0.00036	0.00036	0.00036	0.00036	0.00036	0.00036	0.00036
60	0.00024	0.00024	0.00024	0.00024	0.00024	0.00024	0.00024	0.00024	0.00024
65	0.00002	0.00002	0.00002	0.00002	0.00002	0.00002	0.00002	0.00002	0.00002
70	-0.00019	-0.00019	-0.00019	-0.00019	-0.00019	-0.00019	-0.00019	-0.00019	-0.00019
75	-0.00048	-0.00048	-0.00048	-0.00048	-0.00048	-0.00048	-0.00048	-0.00048	-0.00048
80	-0.00074	-0.00074	-0.00074	-0.00074	-0.00074	-0.00074	-0.00074	-0.00074	-0.00074
85	-0.00094	-0.00094	-0.00094	-0.00094	-0.00094	-0.00094	-0.00094	-0.00094	-0.00094
90	-0.00108	-0.00108	-0.00108	-0.00108	-0.00108	-0.00108	-0.00108	-0.00108	-0.00108

CONFIGURATION B

ALPHA (DEGREES)	C_{y_p} COEF 16	C_{ℓ_p} COEF 17	C_{n_p} COEF 18	C_{y_r} COEF 19	C_{ℓ_r} COEF 20	C_{n_r} COEF 21	C_{z_q} COEF 22	C_{m_q} COEF 23	C_{x_q} COEF 24
0	0.0	-0.15000	0.02000	0.0	0.20000	-0.19000	0.0	-1.10000	0.0
5	0.0	-0.17000	0.02000	0.0	0.29000	-0.20000	0.0	-1.10000	0.0
10	0.0	-0.19000	0.02000	0.0	0.40000	-0.21200	0.0	-1.10000	0.0
15	0.0	-0.21500	0.03000	0.0	0.55000	-0.23500	0.0	-1.10000	0.0
20	0.0	-0.25000	0.05800	0.0	0.75000	-0.28000	0.0	-1.10000	0.0
25	0.0	-0.29000	0.06000	0.0	0.90000	-0.37000	0.0	-1.10000	0.0
30	0.0	-0.32000	0.00100	0.0	0.54000	-0.54000	0.0	-1.10000	0.0
35	0.0	-0.29000	-0.12400	0.0	0.40000	-0.51700	0.0	-1.10000	0.0
40	0.0	-0.22500	-0.02100	0.0	0.30000	-0.45000	0.0	-1.10000	0.0
45	0.0	-0.18200	0.12000	0.0	0.22000	-0.35000	0.0	-1.10000	0.0
50	0.0	-0.15500	0.15000	0.0	0.10000	-0.24000	0.0	-1.10000	0.0
55	0.0	-0.13200	0.18000	0.0	0.05000	-0.17000	0.0	-1.10000	0.0
60	0.0	-0.11700	0.22000	0.0	0.0	-0.12000	0.0	-1.10000	0.0
65	0.0	-0.11000	0.16000	0.0	0.0	-0.08000	0.0	-1.10000	0.0
70	0.0	-0.11000	0.05000	0.0	0.0	-0.06000	0.0	-1.10000	0.0
75	0.0	-0.11000	0.0	0.0	0.0	-0.06000	0.0	-1.10000	0.0
80	0.0	-0.12000	0.0	0.0	0.0	-0.08000	0.0	-1.10000	0.0
85	0.0	-0.12800	0.05000	0.0	0.0	-0.05000	0.0	-1.10000	0.0
90	0.0	-0.13500	0.14000	0.0	0.0	-0.04400	0.0	-1.10000	0.0

REFERENCES

1. Buehler, R. R., The Derivation, Solution, and Analysis of Airplane Spin Equations Modeled in an Inertial Coordinate System, Masters Thesis, United States Naval Postgraduate School, Monterey, California, March 1972.

2. Champoux, R. L., The Numerical Solution and Analysis of Airplane Spin Equations Modeled in an Inertial Coordinate System, Masters Thesis, United States Naval Postgraduate School, Monterey, California, December 1972.

3. Etkin, B., Dynamics of Flight, 1st ed., Wiley, 1959.

4. National Aeronautics and Space Administration TN D-3416, A Study to Determine Effects of Applying Thrust on Recovery from Incipient and Developed Spins for Four Airplane Configurations, by S. B. Grafton, 1966.

5. National Aeronautics and Space Administration TN D-6670, Investigation of an Automatic Spin-Prevention System for Fighter Airplanes, by W. P. Gilbert and C. E. Libbey, 1972.

6. National Aeronautics and Space Administration TN D-6978, An Engineering Optimization Method with Application to STOL-Aircraft Approach and Landing Trajectories, by H. G. Jacob, 1972.

7. National Aeronautics and Space Administration TN D-6926, Analytic Prediction of Airplane Equilibrium Spin Characteristics, by W. D. Adams, Jr., 1972.

8. National Aeronautics and Space Administration TN D-2243, Effects of Aircraft Relative Density on Spin and Recovery Characteristics of Some Current Configurations, by W. D. Grantham and S. B. Grafton, 1965.

9. National Aeronautics and Space Administration TN D-2191, Analytical Study of Aircraft-Developed Spins and Determination of Moments Required for Satisfactory Spin Recovery, by E. L. Anglin and S. H. Scher, 1964.

10. Greenwood, D. T., Principles of Dynamics, 1st ed., Prentice-Hall, 1963.

INITIAL DISTRIBUTION LIST

		No. Copies
1.	Defense Documentation Center Cameron Station Alexandria, Virginia 22314	2
2.	Mr. D. S. Hague Aerophysics Research Corporation Cypress Point Apartments, #262 Cypress Point Drive Palo Alto, California	1
3.	Library, Code 0212 Naval Postgraduate School Monterey, California 93940	2
4.	Chairman, Department of Aeronautics Naval Postgraduate School Monterey, California 93940	1
5.	Professor M. H. Redlin, Code 57Rp Department of Aeronautics Naval Postgraduate School Monterey, California 93940	2
6.	Professor L. V. Schmidt, Code 57Sx Department of Aeronautics Naval Postgraduate School Monterey, California 93940	1
7.	Lieutenant S. T. Keith, USNR 39 Milton Road North Quincy, Massachusetts 02171	2

Security Classification

DOCUMENT CONTROL DATA - R & D

(Security classification of title, body of abstract and indexing annotation must be entered when the overall report is classified)

GINATING ACTIVITY (Corporate author) Naval Postgraduate School Monterey, California 93940	2a. REPORT SECURITY CLASSIFICATION Unclassified 2b. GROUP

ORT TITLE

PREDICTION OF AIRPLANE STEADY SPIN CONDITIONS BY A PARAMETER OPTIMIZATION SCHEME

CRIPTIVE NOTES (Type of report and, inclusive dates)

Master's Thesis; September 1973 --

HOR(S) (First name, middle initial, last name)

Stephen T. Keith, LT., USNR

ORT DATE	7a. TOTAL NO. OF PAGES	7b. NO. OF REFS
September 1973	119	10

NTRACT OR GRANT NO. OJECT NO.	9a. ORIGINATOR'S REPORT NUMBER(S) 9b. OTHER REPORT NO(S) (Any other numbers that may be assigned this report)

STRIBUTION STATEMENT

Approved for public release; distribution unlimited.

PPLEMENTARY NOTES	12. SPONSORING MILITARY ACTIVITY Naval Postgraduate School Monterey, California 93940

STRACT

To aid in the modeling of a steady state spin, the equations of motion of an airplane are formulated in a cylindrical coordinate reference frame. The derivation of the equations is presented and the resulting equations are simplified for the equilibrium spin condition. These simplified equations are used in an unconstrained computer parameter optimization technique that algebraically solves the differential equations for the equilibrium state. The results of the computer work are presented and compared with previous prediction schemes. The potential of the method is demonstrated by application to a study of the effects of density variation.

Security Classification

KEY WORDS	LINK A		LINK B		LINK C	
	ROLE	WT	ROLE	WT	ROLE	WT
Aircraft						
Steady Spin Prediction						
Cylindrical Coordinate Model						
Lagrange Derivation of Equations						
Parameter Optimization						
Computer Prediction Program						
Density Variation Effect						

Thesis 146095
K2526 Keith
c.1
Prediction of airplane
steady spin conditions by
a parameter optimization
scheme.